PRACTICAL GAUGING
Epitomized,
WITH THE
Use, and Construction of the Tables
OF
LOGARITHMS.

Whereunto is added
An
ABRIDGMENT of the LAWS of
EXCISE now in Force.

Intra Spem Veniæ. Hor. de Ar. Poët.

By *William Hunt*, Student in the
MATHEMATIKS, and one of the General
Supervisors for the Duty of *Excise*.

YORK, 1683
Printed, by *J. White*, for the *AUTHOR*. 16[illegible]

Ordered by the Chief Commissioners, and Governors of Excise, that this BOOK (entituled *Practical Gauging Epitomized*) be forthwith Printed and Published.

To the Worshipfull

Sr *Dennis Ashburnham.*
Parry.
Major *Robert Huntington.*
Dr. *Charles D'avenant.*
Col. *John Friend.*
Felix Calverd. &
Nath. Hornbey.
} Esquires

Chief Commissioners, and Governors for the Management of His Majesties Revenue of *EXCISE.*

Throughout the Kingdom of *England*, Dominion of *Wales*, and Town of *Berwick* upon *Tweed.*

William Hunt,

as Duty obliges

Humbly Dedicates to their Patronage this his

Plaital Gauging Epitomized.

To the Reader.

ROugh and Crooked are the wayes that lead to the Practice of Solid Geometry, *and tho many very Learnedly have endeavoured to Conduct the willing thro those* Meanders, *yet few hitherto have arrived at their intended Perfection.*

Tis an eaſie matter to bear the Name, but difficult to be an Expert Gauger.

In this Epitomie *I have given ſuch Directions as may guid the Induſtrious thro many of thoſe Intricacies;*

The Subject *carries its own Evidence, and the* Collections *I willingly ſubmit to the Examination of the more skilfull* Geometritian, *hopeing that they will not deſerve his ill opinion, but rather be an occaſion to invite him to the Edition of ſome things of this nature of more worth and acuracy;*

The Critick, *who condemns without juſt conſiderations I leave to the* Poët,

——————— Invide, quid laceras,
Dum bona ſi non ſunt, tu meliora doce.

So wiſhing they may be as kindly received as they were really Intended, I remain

Ripon May, the 10th 1683.

A true Lover of all that are *Mathematically* enclined.

William Hunt.

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CHAP.

Of Decimal Arithmetick, which is an easy kind of Fractions: And if all the Accounts of Money, Weights, Measures &c. were divided and subdivided into Tens, Then all the tedious Reductions in vulgar Fractions might be avoided.

I. Notation of Decimals.

1. *Note. A Decimal Fraction* is such an one whose *Numerator* only is expressed, the *Denominator* being understood, and is an Unit with as many Cyphers postponed as there are Figures in the *Numerator*; thus the *Denominator* of .5 is 10; of .15 is 100; of. 158 is 1000 and so forwards.

2. *Note. A Decimal Fraction* is not written in a smaller Figure as the Vulgar, but in the same Line and Figure, as the Integer or whole Number, and is distinguished by seperating the Fraction by a Point, Comma or other mark; So 1. $\frac{5}{10}$ is written thus 1 . 5; or 1, 5; or $1\frac{5}{10}$;

3. *Note.* When the *Numerator of a Decimal Fraction* consists not of as many places as the *Denominator* hath Cyphers, prepone as many Cyphers as will make up the Number; so $\frac{5}{100}$ is written thus .05; $\frac{15}{1000}$ thus .015; $\frac{158}{10000}$ thus .0158.

4. *Note. Cyphers* at the end of a *Decimal Fraction* do neither encrease, nor decrease the value thereof, therefore when the *Numerator* and *Denominator* do each end with a *Cypher or Cyphers*, seperate equal Cyphers in both, so will the Fraction be reduced into lesser Terms, thus. $\frac{20}{100}$, $\frac{200}{1000}$, $\frac{2000}{10000}$, may be reduced into $\frac{2}{10}$.

5. *Note.* *Decimal Fractions* may be reduced to one common *Denominator*, by postponing as many Cyphers to such *Numerators* as are defective, as will make them to consist of the same Number of Places, thus. 02. 03. 025, may be reduced to. 020, .030, .025, which have a 1000. for a common *Denominator*.

6. *Note.* *Cyphers* postponed to any Number in *Vulgar Arithmetick* increase it Tenfold; and preponed to any Number decreaseat in *Decimals* Ten-fold; thus 17 with a Cypher postponed is 170; but with a Cypher preponed it is. .017 *(that is)* seaventeen parts of a Thousand.

II. Addition of Decimals.

In *Addition of Decimals*, place every number under its proper *Denominator (that is)* whole Numbers under whole Numbers, Tenths under Tenths, and so every respective Figure under the same estimation with that above it, seperating the whole Numbers from the Fraction, and adding them together as whole Numbers, still setting down the Excess above Ten, and carrying the Tenths to the next place towards the left hand.

25. 725	13. 56981	32. 056
46. 6375	44. 546	7. 07
22. 9421	17. 25007	.9
106. 3046	75. 36388	40. 026

III. Subduction of Decimals.

In *Subduction of Decimals*, observe the same order in placing the given Numbers as in Addition, and then Subduct the Lesser from the Greater as in whole Numbers.

.55	24. 04338	37
.04333	. 65	.104
.50667	23. 39338	36.896

IV. Mul-

IV. Multiplication of Decimals.

In *Multiplication of Decimals*, Multiply the Numbers given as if they were whole Numbers, then ſeparate as many Figures from the Product towards the Right hand as there are *Decimals* both in the *Multiplicand, and Multiplyer*; which Figures ſo ſeperated are the *Fraction* of the Product, & the Figures beyond the Point towards the Left hand the *whole Numbers*.

7. *Note*. If the *Product* doe not conſiſt of as many places as there are Decimals in both Multiplyers *(which may happen when the Product is a Fraction)* ſupply that Defect by prepoſning as many Cyphers as will make up the Number.

8. *Note*. In Multiplication of *Whole Numbers* the Product is alwayes *encreaſed* ſo many times more then the Multiplicand, as the Multiplyer containes Units, as 5 times 3 makes 15.

But in Multiplication of *Fractions*, the Product is alwayes *decreaſed* and made leſs then either the *Multiplicand, or Multiplyer alone*;

The Reaſon is becauſe 1, being multiplyed by 1, can produce but 1, therefore that which is leſs then 1, being multiplyed by that which is leſs then 1, muſt needs be decreaſed by the *Multiplication*. And this Decreaſe bears the ſame proportion to the *Multiplyer*, as the *Multiplicand* bears to the Unit whereof it is apart as in *exam*. 3.

1. 305	246. 25	.75
56.3	35	.0125
73.4715	8618. 75	.009375

V. Diviſion of Decimals.

In Diviſion of Decimals to find the Value of the Quote, Obſerve this general Rule.

There muſt be ſo many Figures ſeparated from the Quote, to-

ward the Right hand, as will make the Decimals in the Diviſor (if any) equal to thoſe in the Dividend.

9. *Note.* In Diviſion of Decimals the *Dividend* muſt ſometimes be prepared by poſtponing Cyphers ſufficient, that it may have more *Decimals* then are in the *Diviſor.*

10. *Note.* If the *Decimals* in the *Diviſor* and *Quote* amount not to thoſe in the *Dividend* ſupply that Defect by preponing as many Cyphers as will make them equal.

Exa. Let the Dividend be 82, *and the Diviſor*, .056.

To the *Dividend* poſtpone a competent numbers of Cyphers to make room for the *Diviſor*, and the work will ſtand thus——

.056) 82. 00000

Then Enquire how many times 56 in 82.[2] you will have it once, place 1 in the Quote, and multiply the Diviſor by 1, (*beginning alwayes at the place of Unity*) ſaying once 6 is 6, and placing it under 2, again once 5 is 5, place 5 under 8, and draw a Line, Laſtly ſubduct 56 from 82, and there reſts 26 for a new Dividend, to which poſtpone the next Figure in the Dividend (*here a Cypher*) and it will ſtand thus

.056) 82. 00000 (1
56·
260

Repeat the ſame work again, and ſay, how many times 56 in 260? you will find 4 times, place 4 in the Quote, and Multiply the Diviſor 56 by 4, ſaying four times 6 is 24, ſet 4 under 0, and carry 2, four times 5 is 20 and 2 you carryed is 22, which placed under 26 ſubduct and bring down the next Figure, and the Ex. will ſtand thus——

.056) 82. 00000 (14
56··
260
224
360

Again ſay how many times 56 in 360? you may have 6 times by which multiply the Diviſor 56, the Product 336 ſubducted from 360 leaves 24 for a new Dividend: ſee the whole operation——

.056

.056) 82. 00000 (1464.28
56 .....
260
224
360
336
240
224
160
112
480
448
32

Now there being 3 *Decimals* in the *Diviſor*, and 5 in the *Dividend*, you muſt ſeperate 2 in the *Quote* to make them equal, ſo the whole Numbers are 1464, and the Fraction 28 according to the *general Rule*.

The *Points* underneath ſerve to inform you how far you have proceeded in your operation, and muſt be removed one place further towards the right hand at every Diviſion, till you come to the laſt figure in the Dividend.

And as long as there is any Remainder if you poſtpone Cyphers to it, and work thus you may have as many *Decimals* as you pleaſe.

And this in my opinion is a very ready way, and not at all burdenſom to the memory.

However becauſe many Able *Arithmetitians* have thought the way of placing the Diviſor under the Dividend, a convenient way for giving a true Value to the Quote, I ſhall Inſert the Rule laid down by them, and leave the *Country Gauger* to follow that which he finds moſt facil.

Rule. The firſt Figure in the Quote will alwayes be of the ſame Denomination with that Figure in the Dividend which at the firſt demand ſtandeth (or is ſuppoſed to ſtand) directly over the place of Unity in the Diviſer.

VI. Re-

VI. Reduction of Decimals.

A Vulgar Fraction may be reduced to a *Decimal* by postponing Cyphers to the *Numerator*, and dividing it by the *Denominator*.

17 Shillings, 1 Penny, 1 Farthing } being { 17/20, 1/240, 1/960 } of a Pound will be found { .58, .0041 667, .0010417

45 Minutes being 45/60 of a Degree will be .75

9 Inches being 9/12 of a Foot will be .75

Observe that the Half the Number of Shillings is the Decimal for Shill.

But it being tedious to reduce *vulgar Fractions* this way, it will be necessary to have Tablets ready prepared of the most notable known parts of Money, Weight, Measure, Time, &c. Two whereof I have inserted in the following Page.

A *Decimal Fraction* may be reduced to a *Vulgar* thus——

Draw a *Perpendicular* Line before the *Decimal Fraction*, and multiply it by the next *lower Denomination* of the Integer, so much of the Product as falleth beyond the Line towards the left hand is the Value in that *Denomination*, and so forwards to the least known parts of the Integer, Keeping the same number of places towards the right hand of the Line, as there are places in the *giving Fraction*.

	.0010417 Decimal of 20 Shillings in a Pound	.75 Dec. of 12 Inc. in a Foot
S. 0	.0208340	Inc 9|00
	12 Pence in a Shill.	
D. 0	2500080	.75 Dec. of 60 Min in a Degree
	4 Farth. in a penny	
F. 1	.0000320	Min. 45 |00

Integer

Integer. 1. *Pound sterling*			
Shill.	Dec.	Pen.	Decimals
19	.95	11	.0458333
18	.90	10	.0416667
17	.85	9	.0375000
16	.80	8	.0333333
15	.75	7	.0291667
14	.70	6	.0250000
13	.65	5	.0208333
12	.60	4	.0166667
11	.55	3	.0125000
10	.50	2	.0083333
9	.45	1	.0041667
8	.40		
7	.35	Far.	
6	.30		
5	.25	3	.0031250
4	.20	2	.0020833
3	.15	1	.0010417
2	.10		
1	.05		

Integer 1 *Foot*	
Inc.	Decimals
11	9166667
10	8333333
9	.7500000
8	.6666667
7	.5833333
6	.5000000
5	.4166667
4	.3333333
3	.2500000
2	.1666667
1	.0833333
Qua.	
3	.0625000
2	.0416667
1	.0208333

The use of the Tablets is,

1. To turn any number of Shillings Pence, Farthings &c. into Decimals, as 15 S. 6 d. $\frac{1}{4}$ will be in Decimals. 7770833

S. 15—.75
d. 6—.025
f. 2—.0020833

Sum—.7770833—15 S. 6d. $\frac{2}{4}$

2. To turn any number of Decimals into the known parts of what they represent whether money &c.
as thus .7770833 reduced will be 15 S. 6d 1;

.7770833 Decimal given,
S15—.75 next gr. subd.
.0270833
d.6—.025 next gr. subd.
f. 2—.0020833.

VII. The Extraction of the Square-Root.

A Square Number has a Side, or Root, which multiplied by its ſelf produces that *Square Number*, as 36 is a Square Number, whoſe Side is 6, for 6 times 6, is 36 as in the following Tablet.

Squares	1	4	9	16	25	36	49	64	81
Roots	1	2	3	4	5	6	7	8	9

But if the given Number exceed any of the nine Digits as 10601536 then it muſt be prepared thus—— ———

Make a Point over the place of *Vnity*, and over every *ſecond Figure* after, the Number of thoſe Points ſhews the Root, or how may Figures will be in the Quote; then——

1. Enquire the greateſt Root of the firſt Point or Square 10, which being no Square Number, take the Root of the next leſſer Square *viz.* 3, which ſet in the Quote, placing the ſquare thereof under 10 the firſt ſquare, and ſubducting it there remains 1 [*This is the firſt work and is no more to be repeated.*]

To the Remainder poſt pone the next ſquare, or the two next Figure 60, it makes 160, which call the Reſolvend.

2. Double the Quote, and it makes 6, which ſet before 160 for a Diviſor [*which is every time to be repeated: And note that the Diviſor when multiplyed muſt not reach by one place ſo far as the place of Vnity in the Reſolvend.*] ſo 6 will be found in 16 twice, ſet 2 in the Quote, and alſo after the Diviſor 6, and multiply 62 by 2, the Product 124 ſubducted from 160 the Reſolvend leaves 36, to which poſtpone the next ſquare 15, it makes 3615 for a new Reſolvend.

3. Double the Quote 32, and it makes 64 for a new Diviſor, and will be found 5 times in 361, ſet 5 in the Quote, and alſo after the Diviſor 64, and multiply 645 by 5, the Product 3225, Subducted from 3615 the Reſolvend leaves 390, to which poſtpone the laſt ſquare 36, and it makes 39036 for a new Reſolvend.

4. Double the Quote 325, and it makes 650, for a new Diviſor, and will be found 6 times in 3903, ſet 6 in the Quote, and alſo after the Diviſor 650, and multiply 6506 by 6, the Product is equal to the Reſolvend.

1 0 6 0 1 5 3 6 (3256 — Root

9

62) 160

124

645) 3615

3225

6506) 39036

39036

Note, when the Product exceeds the Resolvend, or the Remainer exceeds the Divisor a lesser, or greater Figure must be placed in the Quote.

If any Fraction remain postpone a sufficient number of *Pairs of Cyphers* to the Number first given, then esteeming it one entire number work according to the former Rules, and so many Points as were placed over the whole Number, so many whole Numbers will be in the Root, the Rests towards the right hand will be the Numerator of a Decimal Fraction, which will consist of as many places, as there were points over the Cyphers postponed.

VIII. The Extraction of the Cube-Root.

A *Cube Number* has a Side or Root, which multiplied by itself, and that Product again by the Side produces the *Cube*, as 216 is a Cube Number whose Side is 6; for 6 times 6 is 36, and 6 times 36 is 216 as in the annexed Tablet.

Cubes	1	8	27	64	125	216	343	512	729
Roots	1	2	3	4	5	6	7	8	9

But if the given Number exceed any of the nine Digits as 146363183 then it must be prepared thus———

Make a Point over the place of *Unity*, and over every *third Figure* after, the Number of those Points shews the Root, or how many Figures must be in the Quote, then———

1. Enquire what is the greatest Root of the first Point, or Cube 146, which being no Cube number, take the Root of the next lesser Cube *viz.* 5 which set in the Quote, placeing 125 the Cube thereof under 146 the first Cube, and subducting it there remains 21.

[*This is the first work, and is no more to be repeated.*]

To this Remainder postpone, the next Cube, or the three next Figurs 363, it makes 21363 which call the Resolvend.

2. Square the Quote, and Triple the Product, as 5 times 5 is 25, and 3 times 25 is 75, which set before 21363 for a Divisor, [*which must be found at each several operation: And note that the Divisor when multiplied must not reach by two places so far as the place of Vnity in the Resolvend.*] so 75 will be found in 213 twice, set two in the Quote, and multiply 75 by 2, setting the Product 150, two places short of the place of Vnity; next Triple the former figure, and multiply it by the Square of 2 the last Figure in the Quote, thus 3 times 5 is 15, and 4 times 15 is 60, which set one place nearer the place of Vnity; Lastly Cube 2, which is 8, and set it under 3 the place of Vnity, then add these three sums together and they make 15608, which subducted from 21363 there remaines 5755; To which postpone the next Cube or the three next Figures 183, and it makes 5755183 for a new Resolvend.

3. Square the Quote 52, the product is 2704, which Tripled is 8112, and set it before 5755183 for a new Divisor, so 8112 will be found in 57551 seven times, set 7 in the Quote, and multiply 8112 by 7 the Product is 56784 which set two places short of the place of Vnity; next Triple 52 the former Figures in the Quote and multiply it by 49 the Square of 7 the last Figure, it makes 7644 which set one place nearer the place of Vnity; Lastly Cube 7 which is 343, and set it under the place of Vnity, then adding them together they make 5755183 equal to the Resolvend.

Note, when the sum of these three last Numbers exceeds the Resolvend, or the Remainder exceeds the Divisor, the work is erroneous, and a lesser, or greater Figure must be placed in the Quote.

If any Fraction remain postpone a competent number of *Ternaries of Cyphers* to the number first given, then esteeming it one entire Number the extraction is to be made according to the preceding Rules; and so many Points as were placed over the

the whole Number given, so many whole Numbers will be in the Root, the rest towards the Right hand will be the Numerator of a Decimal Fraction which will consist of as many places as there were Points over the Cyphers postponed;

```
        146 363 183 ( 52 = Root
        125
   75 ) 21 363
        150
        ..60
        ....8
        15608
 8112 ) 5755183
        56784
        ..7644
        ....342
        5755183
```

CHAP. II.

Of several Definitions in Geometry, and Propositions for the speedy mensuration of divers Superficies, and Solids.

In *Geometry* there are but *three Dimentions* viz.

A Line which hath only Length.

A Superficies which hath Length and Bredth.

A Solid which hath Length, Bredth, and Depth, or Thickness.

Now when it is known how many Inches &c. are in a Line;

Or how many Square Inches &c. are in a Superficies;

Or how many Cube Inches &c. are in a Solid;

Then is the Content of either of these Magnitudes said to be known.

1 Prop. To find the Content of a Square, or Cube

A Square, is a Figure consisting of four equal Sides, and four right Angles, like any of the faces of a Dye; and a *Cube* is a Solid erected on such a Plain or Base.

1. *By the Side and Depth to find the Content.*

Multiply the Side into self (*which is called Squaring a Number*) the Product is the Area in Inches, which multip. by .0035461, or divided by 282 (*the Cube Inches in an Ale Gallon now in use*) the Product, or Quote is the Content at 1 Inch deep in A. G. and that multip. by the Altitude or Depth produces the whole Content.

Quote of 1, divid. by 282. —— .0035461
Side given 60 Inches Sq. —— 3600
Content at 1 Inch deep in A. G. —— 12. 7659600
Alt, or Depth in Inc. —— 20
Whole Content in A. G. —— 255. 3192000

Applicable for finding the Content of a Square Cooler; as also a Square Tun the Bases, and Sides Above, and Below Equal.

§. *Or By the Table for Gauging Right Lined Figures.*

Against the Side, in the first Column towards the left hand stands a Number, which multip. by the Side produces the Content upon 1 Inch in A. G. seperating 5 figure as in Decimals.

Note, that this Table is made by a continual Addition.

Against 60 the Side in the Tab. stands —— .21277
Multip. by the Side in Inch —— 60
Content upon 1 Inch in A. G. —— 12. 76620

And if the Inches given exceed the Table, take the Number against half which doubled is the Number required.

§. *Or by the Table of Areas of Squares.*

The Tab. Number correspond. to the Side is the content at 1 Inch deep without any more trouble.

60 Inch Side per Table. —— 12. 7660

Note, that the Square of any Side Multiplied by .0035461 or Divided by 282, produces or Quotes the several Numbers in this Tab. but the second Differences being equal, it may more readily be made by Interpolation as directed in my Guide page. 114.

2. *By the Content, and Depth; to find the Side.*

Divide the Content by the Depth, the Quote is the Content at 1 Inch deep, against which in the Table of Areas of Squares stands the Side.

20) 255. 3200 (12. 7660 = 60 Side

3. *By the Content, and Side; to find the Depth.*

Divide the Content by the Area correspond to the Side, the Quote is the Depth.

12. 7660) 255. 3100 (20—Depth.

4. *By the Side of a Square; to find the Diagonal.*

Square the Side, and Double it, the sq. R. of the sum is the Diag. 60 Side. sq. 3600 doubled is 7200 whole sq. R. 84.853 is Diag.

5. *By the Side of a Square, or Cube; to Encrease or Diminish it, according to any Proportion given.*

Square, or Cube the given side, and multiply it by the Proportion desired, the Sq. or Cube R. of the Product is the Side.

10 side sq. —— 100	7 side Cubed —— 343
given Prop. —— 3	given Prop. —— 2
sq. R. 17.32. side. 300	Cube R. 8.819 side. 686

II. Prop. To find the Content of an Oblong, or Parallelopipedon.

An Oblong or Long Square, is a Figure having four right Angles, and four Sides, but not of equal Length, any two of which being opposite are of one Length, and Parallel, like a well Printed page in a Book; And a *Parallelopipedon* is a solid erected on that plain.

1. *By the Sides, and Depth; to Find the Content.*

Multiply the Length by the Bredth, the Product is the Area in Inc. which multiplyed by .0035461, or Divid by 282 gives the Content at 1 Inch deep in A. G. and that multiplied by the Depth produces the whole Content in A. G.

Length. 60 } Inch.
Bredth. 32 }

282) 1920 (6. 8085 A. G.

Depth in Inch. —— —— —20

Whole Content in A. G. —— 136. 1700

Applicable for finding the Content of a Cooler in form of an Oblong, as also an Oblong Tun, the Bases above, and below equal, but unequal Sides at each Base.

§. Or

Or by the Table for Gauging Right-lined Figures.

Multip. the Tabular Number agreeing to either of the Dimentions by the other, the Product is the Content upon 1 Inc. in A. G.

32 Bredth per Tab. ——————————— 11348
Length in Inc. ——————————————— 60
Content at 1 deep in A. G. ——— ——— 6.80880

§. *Or by the Table of Areas of Squares.*

Multip. the Length by the Bredth, the Sq. R. of the Product is the Side of a Square equal to the Area of the Oblong. Or with the Product enter the Table of Squares, and find the neereſt number to it in any of the Angles, then againſt it in the Side *(under Root)* and over it at Top, you have the Inc. and Tenths of the Root, with which enter the Table and take out the Correſp. Area.

32 per 60 = 1920 Sq. R. is 43. 8 ——————— 6.8085 A. G.

2. *By the Content, Depth, and one Side; to find the other.*

Divide the Content by the Depth, the Quote is a Mean Area, which divide by the given Side, with the Quote enter the Table for Gauging Right-Lined Figures and againſt it in the firſt Column towards the left hand ſtands the other Side.

20) 136.17600 (6.80880 Mean Area.
given Side 60) 6.80880 (.11348 = 32 Side ſought.

III. Prop. To find the Content of a Rhombal, or Rhomboidal Solid.

A Rhombus, is a Figure having four equal Sides, but no Right Angles, yet the oppoſite Angles are equal, viz. two Obtuſe, and two Acute, like a Diamond on the Cards, or an ordinary Quarry of Glaſſe.

A Rhomboides is a Figure conſiſting of unequal Sides, and Angles, yet the oppoſite Sides, and Angles are equal.

Let fall a Perpendicular from one of the Obtuſe or Blunt Angles, upon the oppoſite Side, and Multip. them, the Product is the Area in Inc. which multip. by. 003546l, or Divid. by 282, gives the Content upon 1 Inch in A.G. and that Multip. by the Depth produces the whole Content in A. G.

§. Or

§. Or Divide it into two Triangles by a Diagonal drawn between either pair of Angles, then let fall a Perpend. upon that Diag. from either of the contrary Angles, and Multip. them, the Product is the Area in Inc.

Side of the Rhomb. ——60
Perpend.——————55

282) 3300 (11. 70213 A. G.

Depth in Inc.——————————20
Whole Content in A. G.——234. 04260
Diagonal between the Acute Angles——103. 125
Perp. let fall from the Obtuse Angle——32
Area in Inc. as before——————3300. 000

Applicable for finding the Content of a Back, or Tun in either of these Forms.

§. *Or by the Table for Gauging Right-lined Figures.*

Multip. the Tab. Number corresp. to the Side, or Perpend. by the Contrary Number, the Product is the Content upon 1 Inch in A. G.

55 Inch Perp. per Tab.——————.19504
Side given——————————60

Content upon 1 Inch in A. G.——11.7.0240

§. *Or by the Table of Areas of Squares.*

A Geometrical mean Proportion between the Side, and Perpend. is the Side of a Square equal to the Area of the Rhombus, or Rhomboides, with which enter the Table and take out the corresp. Area.

55 per 60 = 3300 Sq. R. 57. 4 Inc.——11.6835 A. G.

VI. Prop. To find the Content of a Triangle or Triangular Solid.

A *Triangle* is a Figure comprehended of three right lines, including three Angles; And of these according to the Nature and Quantity of their Angles, there are three sorts viz.——

Orthogonium or Rectangled } having { One Right } Ang.
Amblygonium or Obtuse-angled } having { One Obtuse } Ang.
Oxigonium or Acute angled } having { Three Acute }

And these again according to the Length and Proportion of their Sides are distinguished into Seaven viz.

A Rec-

A Rect-angled { Isosceles / Scalenum } Having { Two equal / Two unequal } Sides

An Obtuse-angled { Isosceles / Scalenum } Having { Two equal / Two unequal } Sides

An Acute-angled { Equilator / Isosceles / Scalenum } Having { Three equal / Two equal / Three unequal } Sides

1. *By the Base and Perpendicular; to find the Content.*

Measure the Base, (*which may be any of the Sides, but usually we take the Longest.*) and the Perpend. or neerest distance from that Side to the opposite Angle, and Multip. the whole of the one by half the other the Product is the Area in Inc. which Multip. by .0035461, or divid. by 282 gives the Content upon 1 Inch in A. G. and this last Product multip. by the Depth, produces the whole Content in A. G.

Note that all manner of Triangles are measured by this general Rule.

Or Multip. the Base by the Perpend. half the Product is the Area in Inc.

Multip. for A. G. ——— .0035461

9. 6 Inc. Perp. Multip. by 10 half the Base ——— 96

Content at 1 Inc. deep in A. G. ——— .340256

Depth in Inc. ——— 12

Whole Content in A. G. ——— 4.083072

Applicable for finding the Content of a Back, or Cooler in form of a Triangle, as also a Triangular Tun, the Bases, and Sides Above, and Below equal.

2. *By the Sides without the Perpend: to find the Content.*

From the Semi-perimeter or Half sum of all the Sides, Subduct each Side severally, noteing the three differences; then Multip. the half Sum and these three Differ. continually, the Sq. R. of the Product is the Area in Inc.

The given Sides are { 10, 16, 12 } and the Differ. of each Side from 24 the Semi-perimeter or Half sum are { 4, 8, 12 }

Now 4 Multip by 8 and that again by 12 is = 384, which Multip. by 24 the half Sum, the Product is 9216 whose Sq. R. 96 is the Area.

3. By

3. *By the Sides to find the Perpendicular.*

Square each Side severally, then to the Sq. of the Base, add the Sq. of either of the other Sides; from the Sum Subduct the Sq. of the third Side, and divide half the Remainder by the Base; Lastly Subduct the Sq. of the Quote from the Sq. which before was added to the Sq. of the Base, the Sq. R. of the Remainder is the Perpend.

1. Base 20 } Sq. is { 400 }
2. Side 16 } Sq. is { 256 }
3. Side. 12 } Sq. is { 144 }

20) 256 (12.8

Base Sq. 400
2. Side sq. 256 add
Sum—656
3 Side sq —144 subd.
Rests 512
Half—256

The Second side is——————————256
12. 8 Quote Sq. is——————————163. 84
whose Sq. R. 9. 6. Inc is the Perpend. ——————92. 16

4. *By the Two sides comprehending the right Angle; to find the Hypothenuse, or slope side.*

Square the Sides severally, and add them together, the Sq. R. of the Sum is the Hypoth.

Applicable for finding the Hypoth. of an Isosceles, or Equilater Triangle.

Also the Hypoth. or Slope Side of a Pyramid or Cone-vid Chap. II. Prop. XI. Rule 2. *Also the Diagonal of a Cylindrical Cask.* vid Chap. III. Prop. 1. Rule 4. *Also the Chord of any Segments Arch vid.* Chap. II. Prop. VI. Rule 22.

Perpend of an Isosceles 8 Inc. Sq. = 64
Semi-Base -- 6 Inc. Sq. ——————36
Sq. R. 10 is the Hypoth——————100

5. *By the Hypothenuse, and one Side; to find the other.*

From the Square of the Hypoth. Subduct the Sq. of the given Side, the Sq. R of the Remainder is the third Side.

Applicable for finding the Perpend. and Base of an Isosceles Triangle. Perpend. of an Equilater Triangle.

Also the Altitude, and Diam at Base of a Pyramid, or Cone. vid. Chap. II Prop XI Rules 3. 4.

Also the Diam. and Length of a Cylindrical Cask vid. Chap. III. Prop. 1. Rules 5. 6.

Also the Verſed Sine, and Segments Chord.

Also the Side of a Geometrical Square vid. Chap II. Prop 1. Rule 4.

Hypoth of an Iſoſch. 10 Inc. ſq. —— — —100.

Sem—Baſe 6 Inc. ſq.— —— ——26.

Whoſe Sq. R. 8 is the Perpend.— ——64.

V. Prop. To find the Content of a Trapezial ſolid.

A Trapezium is a Figure conſiſting of four unequal Sides, and Angles. Divide it into Triangles by a *Diagonal* drawn between either pair of oppoſite Angles; & from the other two Angles let fall Perpend. upon the *Diagonal or common Baſe*; then Multip. the whole of the one by half the other, and the Product is the Area in Inc. Or Multip. The Diagonal by the ſum of the Perpend. half the Product is the Area in Inches: Which Multip. by .0035461, or Divided by 282 gives the Content upon 1 Inch in A. G. and that Multip. by the Depth produces the whole Content.

If any Two ſides are Parallel, Multip. the Semi--Sum of the two Paralel ſides by the neareſt diſtance between thoſe ſides, and the Product is the Area in Inc.

Now to take the Dimentions Actually do thus——

Chalk a ſmal Line, and with your Compaſſes or Picker faſten one end in any corner or Angle, ſtretch it out to the oppoſite Angle, and ſtrike a Diagonal Line:

Then Chalk the Line again, and faſtning it at the third Angle, ſtrain it to the Diagon. and get the ſhorteſt diſtance from that Angle, to the Diagon. which is one Perpend.

Doe the ſame from the fourth or laſt Angle to the Diagon. ſo have you the other Perpend.

As for Exam. Let the Diagon. be 120 Inc. one Perpend. 70, and the other 40 Inc. and the Depth 20 Inc. what is the Content in A. G

Multip. for A. G.— —— —.0035461

60 Multip by 90—Area in Inc.— ——5400

Content at 1 Inc. deep in A. G. — ---19.1489400

Depth in Inc.——— —— —— 20

Whole Content in A. G.—— —— 382.9788000

§. *Or by the Table for Gauging Right-Lined Figures.*

Multip. the Tab Number corresp. to the *Base, or sum of the Perpend.* by *half the contrary Number*, the Product is the Area in A. G.

Against 90 sum of Perpend. stands---- ----.31915
Half the Diagon.---- ---- ----60
Content upon 1 Inc. in A. G.---- --.19.14900

§. *Or by the Table of Areas of Squares.*

A Geometrical mean Proportion between the *whole Base*, and *half Sum* of the *Perpendiculars*; Or *Sum of the Perpend.* and *half the Base* is the Side of a Square equal to the Area of the *Trapezium*, against which in the Table stands the Area in A.G.

90 per 60 is 5400 sq. R. is 73. 5---- ---19.1489 A. G.

Note, that to find the Content of any *Multilateral Solid*, you must draw *Diagonals* from Angle to Angle, which will reduce it into *Trapezia's*, *and Triangles*, and then find their Contents severally and sum up all together, which multip. by .0035461, or Divid. by 282 gives the Content at 1 Inch deep in A. G. and that multip. by the Depth produces the true Quantity of A. G. in that irregular Back, or Tun.

In any Right-Lined Figure composed of Triangles, the Triangles are alwayes less by two then the Number of Sides, and the Diagon. are less by three.

VI. Prop, To find the Content of a Circle, and its Segments.

A Circle is a Plain Figure comprehended by one Line, usually described with a pair of Compasses, one Foot remaining fixt, and the other wheeling about makes that Round line, which is called the *Periphery*, *or Circumference*.

The Point exactly in the midle is called the *Center*, from whence all right Lines drawn to the *Periphery* are of equal Length.

The *Diameter is a right Line* drawn thro the Center from one Side of the *Periphery* to the other, dividing the Circle into two equal parts. The Half whereof is called the *Semi-diameter*, or *Radius*. A *Chord*; or *Subtense* is a *Right line* reaching from any part of the *Periphery* to the other, dividing the Circle into two

unequal parts. A *Verſed Sine is a Segment of the Diameter* made by the *Chords* interſecting it at Right-Angles.

Half the Circle is called a *Semi-Circle*; a Quarter a *Quadrant*. A *Segment* is any Portion of a Circle comprehended by the *Periphery*, cut by a Chord not touching the Center.

A *Sector* is a Segment contained inwards under two Right-Angles making an Angle at the Center.

1. *By the Diam, to find the Periph.*

As 7, to 22; Or 70, to 223, or 113 to 355; or 1561, to 4906; Or 1, to 3. 141593; ſo is the Diam. to the Periph.

Periph. of a Circle whoſe Diam. is 1 ——— 3.141593
Diam. in Inc. ——— 20
Periph. is ——— 62.831860

2. *By the Periph; to find the Diam.*

As 22, to 7; Or 223, to 70; Or 355, to 113; or 4906, to 1561; Or 1, to .31831; ſo is the Periph. to the Diam.

Periph. given ——— 62.83186
Diam. of a Circle whoſe Periph is 1; *Or the Quote of* 1, *Divid. by* 3.141593 ——— .31831
Diam. is ——— 20.0000093566

3. *The Diam; to find the Area in Inc. or A. G.*

As 14, to 11; Or 284, to 223; Or 452, to 355; Or 1, to .785398; ſo is the Sq. of the Diam. to the Area in Inc.

Or as 7, to 22; Or 70, to 223; Or 113, to 355; Or 4906. to 1561; Or 1, to 3. 141593; ſo is the Sq. of the Sem-diam. to the Area in Inc. Or Multip. half the Periph. by the Radius, the Product is the Area. Or the Sq. of the Diam. Divided by 1.273239 Quotes the Area in Inc. Again as 1, to .0027851; ſo is the Sq. of the Diam. to the Area in A. G. or the Sq. of the Diam. divid. by 359.0536 Quotes the Area in A. G.

Which is the conſtruction of the Table of Areas of Circles: But the ſecond Differences being Equal, it may be made by an eaſie collection as directed in my Guide p. 114.

Area of a Circle whoſe Diam. is 1 *in Inc. is* ——— .785398
Diam. 20 Inc. Sq. ——— 400
Area in Inc. ——— 314.159200

Quote

Quote of .785398 divid. by 282 is---------.0027851
Diam. 20 Inc. Sq.---------400
Area in A. G.---------1.1140400

4. *By the Area in Inc. or A.G. To find the Diam.*

As 11, to 14; Or 223, to 284; Or 355, to 452; Or 1, to 1.273239; so is the Area in Inc. to the Sq. of the Diam.

Or as 1, to 359.0536 so is the Area in A. G. to the Sq. of the Diam.

Note, that by the Table of Areas you have the Diam. by Inspection.

Sq. of the Diam. of a Circle whose Area is 1 Inc.
Or the Quote of 1. Divid. by. 785398 is— } 1.273239
Area in Inc.— — — — —314. 1592
Sq. R. 20. is the Diam.— — —399.999745 6488
Sq. of 18.9487, the Diam. of a Circle whose Area is 1 A.G. commonly called the Gauge Point— }
Or the Quote of 1, divid. by. .0027851,— } 359.0536
Or the Product of 1.273239 Multip by 282.— }
Area in A. G.— — — — — —1. 11404
Sq. R. 20 is the Diam.— — — —400.000072544

5. *By the Periph to find the Area in Inc. or A.G.*

As 88, to 7; Or 892, to 71; Or 1420, to 113; Or 1, to .079577; so is the Sq. of the Periph. to the Area in Inc.

Or as 1, to .00028219 so is the Sq. of the Periph. to the Area in A.G. Periph. 62. 83186 Sq. is— —3947.8426 &c.

Quote of .785398 divid. by 9.8696 the Sq. of 3. 14. 5927— } .079577
Or the Area of the Circle whose Periph. is 1 in Inc. }
Area in Inc.— — — — —314. 157470 5802
Periph. 62.83186 Sq. is— — 3947. 8426
Quot of .079577 divid. by 282— — — }
Or the Area of a Circle whose Periph. is 1 in A.G. } .00028219
Area in A. G.— — — —1. 11404 1703294

6. *By the Area in Inc. or A.G. to find the Periph.*

As 7, to 88; Or 71, to 892; Or 113, to 1420; Or 1, to 12.56637; so is the Area, to the Sq. of the Periph. in Inc.

Or

Or as 1, to 3543.711; So is the Area to the Sq. of the Periph.
in A G. Area in A.G. —— —— —— 1. 1 14042
Quote of 1, divid. by .00028219 —— —— 3543. 711
Sq. R. 62. 83186 is the Periph —— —— —— 3947. 84288 9862
Area given in Inc —— —— —— 314. 15927
Sq. of 3. 544908, the Periph. of a Circle whose Area is 1, } 12. 56637
Or the Quote of 1 divid. by .079577 ——
Sq. R. 62. 83186 is the Periph. —— —— —— .3947. 84162 57499

7. *By the Diam; to find the Side of a Square Equal.*

As 1, to .886227; So is the Diam; to the Side of a Sq. Equal.
(*which is the Squaring of a Circle.*)
Side of a Sq. equal to a Circle whose Diam. is 1, } .886227
Or the Quote of 1. *divid. by* 1.128379; ——
Diam given in Inc. —— —— —— 20.
Side of the Sq. Equal is —— —— —— 17.724540

8. *By the Side of a Square Equal; to find the Diam.*

As 1, to 1.128379; So is the Side to the Diam.
Diam. of a Circle whose Area is 1, } 1.128379
Or the Quote of 1, *divid. by* .886227 }
Side given in Inc. —— —— —— 17.72454
Diam is —— —— —— 19.99999872066

9 *By the Diam; to find the Side of a Square Inscribed.*

As 1, to .707 : 067; So is the Diam. to the Side.
Or the Sq. R of half the Sq. of the Diam. is the Side.
Side of a Sq. inscribed in a Circle whose Diam. is 1. —— .707106
Diam. given in Inc. —— —— —— 20
Side of the Sq. Inscribed —— —— —— 14.1421340

10. *By the Side of a Square Inscribed; to find the Diam.*

As 1, to 1.414214; So is the Side to the Diam.
Quote of 1, *divid. by* .7071067 *is* —— —— 1.414214
Side Inscribed —— —— —— 4.142134
Diam. is —— —— —— 0.000003892676

11. *By the Periph; to find the Side of a Square Equal.*

As 1, to .282095; So is the Periph. to the Side.

Side

Periph given ——— 62.83186
Side of a Sq equal to a Circle whose Periph. is 1, *Of the Quote of* 1, *divided by* 3.544908; ——— .282095
Side of the Sq. Equal ——— 17.72455354670

12. *By the Side of a Square Equal to find the Periph.*

As 1, to 3.544908; So is the Side, to the Periph.
Periph of a Circle whose Area is 1, *Or the Quote of* 1, *divided by*.282095 ——— 3.544908
Side of the Square Equal. ——— 17.72455
Periph. is ——— 62.83189909140

13. *By the Periph; to find the Side of a Square Inscribed.*

As 1, to.225079; So is the Periph. to the Side.
Periph. given ——— 62.83186
Side of a Sq. Inscribed in a Circle whose Periph. is 1, .225079
Side of the Sq. Inscribed. ——— 14.142132221694

14. *By the Side of a Square Inscribed; to find the Periph.*

As 1, to 4.44285; So is the Side, to the Periph.
Side Inscribed. ——— 14.142132
Quote of 1, *Divid. by*.225079, *is* ——— 4.44286
Periph. is ——— 62.83151257752

15. *By the Side of a Square Equal, to find the Area.*

Square the Side (*that is*) multip. the Side into its self the Product is the Area as in Prop 1. Chap. II..
Side given ——— 17.72454
Side ——— 17.72454
Area in Inc. ——— 314.1593182116

16. *By the Side of a Square Equal; to find the Square Inscribed.*

As 1, to.797884; So is the Side of the Sq. Equal, to the Side of the Square Inscribed
Side of the Square Equal. ——— 17.72454
Multip. proper. ——— .797884
Side Inscribed. ——— 14.142126873336

17. *By the Side of a Sq. Inscribed; to find the Area.*

As 1, to 1.570798; So is the Sq. of the Side Inscribed, to the Area.

Side

Side 14.14213 Sq. ——— 199.99984
Sq. of 1.253315. ——— 1.570798
Area in Inc. ——— 314.159348 68132

18. *By the Side of a Sq. Inscribed; to find the Side of the Sq. Equal.*

As 1, to 1.253315; So is the Side of the Square Inscribed, to the Side of the Square Equal.

Side of the Sq. Inscribed. ——— 14.14213
Quote of 1, *Divid. by* .797884. ——— 1.253315
Side of the Square Equal. ——— 17.724 43660 95

19. *By the Area; to find the Side of a Square Equal.*

The Square Root of the Area is the Side of a Sq. equal to the same Circle. Area of a Circle whose Diam. is 20 — 314.1593
Sq. R. 17.72454 is the Side.

20. *By the Area to find the Side of a Square Inscribed.*

As 1, to .797884; So is the Sq. R. of the Area to the Side Inscribed.

Area 314.1593 whose Sq. R. is ——— 17.72454
Multip. proper. ——— .797884
Side Inscribed. ——— 14.14212687336

21. *By the Chord and Versed Sine; to find the Diam.*

Divide the Sq. of the Semi-Chord, by the Versed Sine, the Quote is the Remainder of the Diam. to which add the Versed Sine, the Sum is the Diam.

Semi-Chord 6 Sq. is ——— 36
Divid. by the Versed-Sine 2 Quotes ——— 18
To which add the Ver. Sine. ——— 2
Sum is the Diam. ——— 20

22. *By the Diam and Versed Sine; to find the Chord.*

From the Diam. Subduct the Versed Sine, and Multip. the Remainder by the Versed Sine, the Sq. R. of the Product is the Semi-Chord.

Diam. given. ——— 20
Vers. Sine Subd. ——— Rest ——— 2
Multip. by V.S. ——— 18
Sq. R. 6. is the Semi. Chord. ——— 36

13. By

23. *By the Diam. and Chord; to find the Verſed Sine.*

From the Sq. of the Semi-Diam. Subduct the Sq. of the Semi-Chord, the Sq. R. of the Remainder Subducted from the Semi diam. leaves the Verſed Sine.

Semi-diam. 10 Inch Sq. — — — 100
Semi-chord. 6 Sq. — — — — 36
Whoſe Sq. R. is 8. — — — — 64
Subd. from — 10 the ſemi-diam.
Reſts V. S. — 2

Note that by theſe three laſt Rules you may find the Axe of a Sphere, as alſo the diam. at Baſe, & Altitude of any Segment thereof.

24. *By the Diam. and Arch Line of any Portion of a Circle which hath One Angle at the Center, to find the Area.*

Multip. the Semi-diam. or Radius by half the Compaſs, or Length of the Arch Line, the Product is the Area.

Half the Arch Line of a ſemi-circle is — — — 15.707965
Semi-diam in Inc. — — — — 10
Area of the Semi-Circle — — — 157.079650
Half the Arch Line of a Quadrant is. — — — 7.853982 5
Semi-diam. — — — — 10
Area of the Quadrant — — — — 78.539825 0

25. *By the Diam. of a Circle, and Verſed Sine; to find the Area of the Segment per Tab. of Segments.*

To the *Verſed Sine*, add three Cyphers, and divide it by the *Diam.* with the Quote enter the Table under V. S. and againſt it ſtands a *ſegment* by which Multip. the Area of the whole Circle. (*found by Rule* 3) the product is the *Segment*.

V. S.

Diam. 20) 2. 000 (100 = V.S.
Area of a Circle whoſe Diam. is 20 is — — — 314.15927
100 V. S. per Tab gives — — — — 05 0
The Segm. is. — — — — 16.33028 2040

26. *By the Diam. of a Circle, and Area of its Segment; to find the Verſed Sine. per Tab.*

To the *Area of the Segm.* add four Cyphers, and divide it by the Area of the whole Circle (*found by Rule* 3) with the Quote enter the Table under *Segm.* and againſt it towards the Left-hand ſtands

ſtands a *Verſed Sine*, which Multip. by the Diam. the Product is the V.S. vid. Chap. III. Prop. II. Rule 2.

314.15927) 16.336282040 (.0520 = Segm.

Againſt .0520 ſtands ——— — ———100 V.S.
Multip. by the whole Diam. ——— — ——20
Verſed Sine is. ——— — ———2.000

27. *By the Diam. of a Circle, and Area of its Segment; to find the Chord.*

By Rule 26 find the Verſed Sine, and by Rule 22. find the Chord, an exam. is needleſs.

28. *By the Chord, and Verſed Sine; to find the Area of the Segm.*

To the Sq. of the Segments ſemi-Chord, add the ſq. of the Verſed ſine, the ſq. R. of the ſum is the Chord of the Half ſegments Arch, which double, and ſubduct from it the Chord of the ſegment; One third part of the difference added to the doubled Chord of the Half ſegments Arch, is the Length of the Arch Line of the whole Segment.

Then *(by Rule 21)* find the Diam. And *(by Rule 24)* find the Area of the doubled Sector; Laſtly *(by Prop. 4. Chap. II.)* find the Area of the doubled Triangle, which ſubducted from the Sector leaves the Area of the *Segment*.

ſemi-Chord 6 ſq. is ——— ———36
Verſed ſine 2 ſq. is——— — ———4

ſq. R. 6.324555 is the Chord of Half the Segm. Arch40
Doubled Chord of Half the ſegm. Arch.———12.65
Chord of the ſegm.——— — ———12.

Differ.———.65
One third is—.217

Doubled Chord of Half the ſegm. Arch———12.65
Sum is the Length of the Arch-Line ——— 12.867
The Half is——— ——— ———6.4335
Multip. by the ſemi-diam.——— — ———10

Area of the Doubled Sector.——— ———64.3350
Semi-chord 6 by 8 the Perpend. = Area of doub. Tria. 48. ſubd.
Reſts the Area of the ſegment ——— — 16.3350

§. Mr.

§. Mr. *Phillips* in his *Purchasers Pattern* p. 125 directs to Multip. the Whole of the One by two thirds of the other, and the Product is the Area.

But a neerer Approximation is to Multip. the Chord by Two thirds of the Versed Sine more .028.

V.S. 2, whose $\frac{2}{3}$ is 1.33334 .028 ——— 1.3613

Chord Line ——— ——— ——— 12

Area of the Segment *propè* ——— ——— 16.3356

29. *The Construction of the Table of Areas of Segments of a Circle*, as I received it from Mr. *Henry Coley an approved Artist*. is comprehended in the three following Precepts, and is the most facil way of making it I have yet seen; being the same Table published by *Adrianus Metius* to three places of Decimals in his Pract. Geom. part II. Chap. 7. p. 216 which he saith was first calculated by *SYBRAND HANS* his words in p. 211 'run thus: *Generalem & satis exactam mensurandi pragmatiam* 'beneficio Tabellæ, *quæ ad omnis generis Doliase extendit, bre-* 'vissimo scripto divulgavit *Magister Sibrandus Hansonis Har-* 'linganus *Ludi magister & Geometria insignis apud Amstel-* 'rodamenses.

1. To the Logar. of the Compl. of the Versed Sine to 500, add this Logar. 9.301030, the Sum is the Co-sine of the Sem-Angle at Center.

2. To the sine, and Co-sine of the Sem-Angle at Center in Degrees, and Decimal Minutes, add this Logar. 1.502850, the sum is the Logar. of the doubled Triangle,

3. To the Logar. of the whole Angle at the Center in Degrees and Decimal Minutes add this Logar. 9.443697 the sum is the Logar. of the doubled Sector.

From which subduct the doubled Triangle the Remainder is the Segment,

Note that all the operation must be Decimal Fractions tho wrought in whole Numbers as is common, and notwithstanding they have an Index preponed, yet they must be supposed Defective. Let the supposed Versed Sine be.263, what is the Segment corresp.

1 Compl, of 263 to .500 the Supposed Rad is 237 Log. 0,374748
Arith. Compl, of the Logar, of .500 is, ——— 9.301030
Co-Sine Sem-Angle at Center $6^{\circ}_{1}.7^{1}_{0}$ ——— —9.675778
2 Sine } Sem-Angle at Center { $6^{\circ}_{1}.7^{1}_{0}$ ——— 9.944718
Co-Sine } { $6^{\circ}_{1}.7^{1}_{0}$ ——— 9.675778
Arith, Comp, of Log, of Area of a Circle whose Dia, is. 1.1,502850
Logar, of the Doubled Triangle, 13284 ——— —1. 123346
3, Whole Angle at Center $12^{\circ}_{3}.40$ — —Log, 2,091316
Arith Compl, of the Logar, of 36°_{0}, ——— —9,443697
Log, of the Doubled Sector 34279, ——— —1. 535013
Doubled Triangle Subd, —— 13284
Rests—20995 the segm, as per Tab,

To turn Sexagessimal Minutes in Decimals, and the contrary, vid, Reduction of Decimals Chap, I, p, 5,

VII. Prop. To find the Content of a Sphere, and its Segment.

A Sphere or Globe is a solid included in one Round superficies, from the Center whereof all Right Lines extended to the superficies are Equal, and is made by the Revolution of a semi-Circle about the Diam,

Any *Diam*, whereof is commonly called the *Axe*,

1, *By the Axe ; to find the Periph,*

The Rule is the same with the I, in Prop, VI, Chap, II,

2, *By the Periph ; to find the Axe,*

The Rule is the same with the 2, in Prop, VI, Chap, II,

3 *By the Axe ; to find the Area in Inches or A, G,*

As 7, to 22; Or 70, to 223; Or 113, to 355; Or 1561, to 4906; Or 1, to 3, 141593; so is the sq, of the Axe to the Area in Inc, Or the Area of the Circle Multip, by 4 produces the Area in Inc, Or Multip, the Axe, by the Periph, the Product is the Area in Inc, Or as 1, to 3, 11404; so is the sq, of the Axe to the Area in A, G,

Area of a sphere whose Axe is 1, in inches ——— 3,141593
20 Inch Axe squared ——— ——— ———400
Area in Inches ——— ——— —1256,637200

Quote

Quote of 3.141593 *divid, by* 282, ——— } —,0111404
Or the Area of a ſphere whoſe Axe is 1, *in A, G,* }
20 Inch the Axe ſquared ——— ——— ——— 400
Area in A, G, ——— ——— ——— 4,4561600

4, *By the Area in Inches, or A, G, to find the Axe,*

As 22, to 7; Or 223, to 70; Or 355, to 113; Or 4906, to 1561;
Or 1, to .31831; ſo is the Area in Inch to the ſquare of the Axe
Or as 1, to 89,7634; ſo is the Area in A, G, to the ſquare of
the Axe, Area in Inch is ——— — — 1256,6372
Quote of 1 *divid, by* 3, 141593, ——— ——— —,31831
ſquare R, 20 is the Axe ——— — — 400,000187132
Area in A, G, ——— ——— ——— 4,45616
Quote of 1, *divid*, by ,0111404 ——— —— 89,7624
ſq, R, 20 is the Axe, ——— ——— 400,090072544

5, *By the Periph; to find the Area in Inc, or A, G,*

As 22, to 7; Or 223, to 70; Or 355, to 113; Or 4906, to 1561;
Or 1, to, 31831; ſo is the ſq, of the Periph, to the Area in Inc,
Or 1, to ,0011 2876; ſo is the ſq, of Perip, to Area in A, G,
62,83186 Periph, ſq, ——— ——— 3947,84263 &c,
Diam, of a Circle whoſe Periph is 1, ——— ,31831
Area in Inches ——— — ——— 1256.637778 &c.
Periph. Sq. ——— — ——— 3947.84263
Quote of .31831 *divid, by* 282, ——— — } ,00112876
Or the Diam, of a Circle whoſe Periph is 1 *in A, G,* }
Area in A, G, ——— ——— 4,4561648131763

6, *By the Area in Inc, or A, G; to find the Periph,*

As 7, to 22; Or 70, 223; Or 113, to 355; Or 1561, to 4906;
Or 1, to 3, 141593; ſo is the Area in Inc, to the ſq, of the
Periph, Or 1, to 885,9279; ſo is the Area in A, G, to the ſq,
of the Periph,
Area given in Inches ——— ——— 1256,6372
Periph of a Circle whoſe Diam, is 1, ——— 3.141592
Sq, R, 62,83186 *is the Periph,* ——— 3947,84261,10596
Area in A, G, ——— ——— ——— 4,456167
Quote of 1, *divid, by,* 00112876, ——— 885,9279
Sq, R, 62,83186 is the Periph, ——— 3947,84267 23593

7. *By*

7. *By the Axe; to Find the solid Content in Inc. or A. G.*

As 21 to 11, Or 426, to 223; Or 678, to 355; Or 1, to .523598; so is the Cube of the Axe to the Solid Content in Inc.

Or as 1, to .0018567; so is the Cube of the Axe to the Solid Content in A. G.

Solid Content of a Sphere whose Area is 1, *in Inc.*—.523598
Axe 20 Inc. Cubed ——— ——— 8000

Content in A. G.——— —— ———4188. 784000
Solid Content of a Sphere whose Axe is 1, *in A. G.*—.0018567
Axe 20 Inc. Cubd.— —— —— —— 8000

Content in A. G.—— —— —— —14. 8536000

8. *Or having the Axe and the Area of the Sphere*; Multip. the whole of the one, by one Sixth part of the other, the product is is the Content.

Area of the Sphere 1256. 6372 whose one Sixth is —209.4395
Axe giving in Inc. — — —20

Content in Inc. ——— ———4188. 7900
Area of the Sphere 4. 45616 A. G. the one Sixth is —— .74269
Axe given in Inc.——— ———20

Solid Content in A. G.——— ———14. 85380

9. *Or having the Axe and Area of the Coresp. Circle*; Multip. the whole of the one, by two thirds of the other, the Product is the solidity.

Area equal to 20 Axe is 314. 1593 whose $\frac{2}{3}$ is —— 209. 4395
Axe given in Inc.———20

Solid Content in Inc. ——— ———4188.7900
20 the Axe per Table gives 1. 1140 where $\frac{2}{3}$ is ——— .7427
Axe given——— ———20

Content in A. G. ——— ———14. 8540

8. *By the Solid Content in Inc. or A. G. to find the Axe.*

As 11, to 21, Or 223. to 426; Or 355, to 678; Or 1, to 1.909855; so is the Content in Inc. to the Cube of the Axe.

Or as 1, to 538.58; so is the Content in A. G. to the Cube of the Axe.

Quote

Quote of 1, Divided by .523598 ——— 1.509855
Content in Inch. ——— ——— 4188. 8
Cube R. 20 is the Axe. ——— 8000. 0006240
Quote of 1, Divided by .0018567 ——— 538. 58
Content in A. G. ——— ——— 14. 854
Cube Root 20 is the Axe. ——— —8000. 06732

9. *By the Periph; to find the Solid Content in Inc. or A. G.*

As 2904, to 49; Or 1, to .016887; so is the Cube of the Periph. to the Content in Inc.

Or as 1, to .000059883; So is the Periph. Cubed, to the Content in A.G. Periph. 62.83186 Cubed is—248050.2955
Solid Content of a Sphere whose Periph is 1, in Inch .016887
Content in Inches ——— ——— 4188.825340101 5
Periph, Cubed ——— ——— —248050.2955
Solid Content of a sphere whose Periph is 1, in A.G. .000059883
Content in A. G. ——— ——— 14.85399584542 65

10. *By the Solid Content in Inc. or A. G; to find the Periph.*

As 49, to 2904; Or 1, to 59. 21715; So is the Content in Inc. to the Cube of the Periph.

Or as 1, to 16699. 23; so is Cont. in A, G, to the Cube of the Periph.
Quote of 1, divid. by .016887. ——— — 59.21715
Content in Inc, ——— ——— ——— 4188, 8
Cube R, 62, 83186 is the Periph, ——— 248050.27834875
Quote of 1, divid. by .000059883. ——— —16699. 23
Content in A, G, ——— ——— 14. 854
Cube R, 62.83186 is the Periph ——— 248050, 36242

11. *By the Axe; to find the Side of a Square equal to the Area.*

As 1, to 1, 772454; so is the Axe, to the side of the Square,
Square Root of the Area of a sphere whose Axe is 1, 1.772454
Axe given in Inches ——— ——— ———20
The side is ——— ——— ———35.449080

21 By

12. *By the Side of a Sq. equal to the Area; to find the Axe.*

As 1, to .56419, so is the Side of the Square to the Axe.
Side of the Square equal—————35.44908
Quote of 1, *divided by* 1.772454 —————.56419
The Axe————— ————— —20.000016445 2

13. *By the Periph; to find the Side of a Square equal to the Area.*

As 1, to, 56419; so is the Periph; to the Side of the Square.
Periph given————— ————62. 83186
Multip. proper————— ————.56419
The side is————— ————. 35.4491070934

14. *By the Side of a Square equal to the Area: to find the Periph.*

As 1, to 1.772454; so is the Side of the Sq. to the Periph.
Side of the Square given————— ————35.4491
Multip.. proper ————— ————1. 77454
The Periph is ————— ————62. 831899014

15. *By the Axe; to find the Side of a Cube equal to the Solid Content.*

As 1, to ,806; so is the Axe to the side of the Cube.
Cube Root of the solidity of a sphere whose Axe is 1,——.806
Axe given in Inches ————. ————20
Side of the Cube is————— ————16. 120

16. *By the Side of a Cube equal to the Solid Content, to find the Axe.*

As 1, to 1. 240695; so is the side of the Cube to the Axe.
Quot of 1, *Divided by* .806 *is* ———— 1. 240695
Side of the Cube, ————— ————16 12
The Axe, ————— ————— 10.00000340

17. *By the Periph; to find the Side of a Cube equal to the Solid content.*

As 1, to .256556; so is the Periph. to the side of the Cube.

Cube

12. *By the Side of a Sq. equal to the Area; to find the Axe.*

As 2, to .56416; ſo is the ſide of the ſquare to the Axe.
Side of the ſquare equal ——— 35.44908
Quote of 1, *divided by* 1.772454 ——— .56419
The Axe. ——— ——— ——— 20.000164452

13. *By the Periph; to find the ſide of a Square equal to the Area.*

As 2, to, 56419; ſo is the Periph. to the ſide of the ſquare.
Periph. given ——— ——— ——— 62.83186
Multip. proper ——— ——— ——— .56419
The ſide is ——— ——— ——— 35.4491070934

14. *By the Side of a Square equal to the Area: to find the Periph.*

As 1, to 1.772454; ſo is the ſide of the ſquare to the Periph
Side of the ſquare given ——— ——— 35.4491
Multip. proper ——— ——— 1.77454
The Periph. is ——— ——— 62.8318990914

15. *By the Axe; to find the Side of a Cube equal to the Solid Content.*

As 1, to .806; ſo is the Axe, to the ſide of the Cube.
Cube R. of the Solidity of a ſphere whoſe Axe is 1. — .806
Axe given in Inches ——— ——— 20
Side of the Cube is ——— ——— 16.120

16. *By the Side of a Cube equal to the Solid Content; to find the Axe.*

As 1, to 1.240695; ſo is the ſide of the Cube to the Axe.
Quote of 1. *divided by* .806 *is* ——— 1.240695
Side of the Cube ——— ——— 16.12
The Axe ——— ——— ——— 20.00000340.

17. *By the Periph; to find the Side of a Cube equal to the Solid Content.*

As 1, to .256556; ſo is the Periph. to the ſide of the Cube

Periph. given—— —— ——62. 83186
Cube R. of the Solidity of a Sphere whose Periph. is 1.—256556
Side of the Cube.—— ——16. 11993067416

18. *By the Side of a Cube equal to the Solidity; to find the Periph.*

As 1 to 3. 897784 ; so is the side of the Cube, to the Periph.
Quote of 1, *Divid. by.* 256556. —— —3.897784
Side of the Cube—— —— ——16. 12
The Periph is—— —— 62.83227808

19. *By the Side of a Cube; to find the Solid Content.*

The Cube of the side is the Content, *vid.* Chap. II. Prop. 1.
Side 16. 12 Cubed is 4188.8 equal to the Content.

20. *By the Solid Content ; to find the Side.*

The Cube Root of the solidity is the side.
Solid Content 4188.8 whose Cube Root is 16.12. is the side.

21. *By the Chord of any Segments Arch ; to find the Area of the Segment.*

As 1, to 3.1415927 ; so is the Sq. of the Chord to the Half Segments Arch, (*found by Prop. VI. Rule* 28) to the Area.
Periph. of a Circle whose Diam. is 1, —— 3.1415927
Chord of half the segments Arch 6.324555 su. is——40.
Area of the Segment —— ——125.6637080

§. Or Multip. the Periph. by the segments Altitude ; the Product is the Area. Periph. given—— ——62. 83186
Segments Altitude.—— —— ——2
Area of the segment.—— ——125. 66372

§. Or as the Whole Axe, to the whole Area, so is the Versed sine, or segments Altitude to the Area of that part.
As 20, the Axe to 1256. 6372 the Spheres Area,
So is 2 the Segments Altitude to 125.66372 the segments Area.

22. *By the Axe and Altitude of any Segm ; to find the Solidity.*

From the Axe, subduct the Altitude, to the Remainder add the Semi-Axe, and Multiply the sum by the Sq. of the Altitude. The Product Multip. by 1. 047197 produces the solid Content in Inches, or by .0037135, for A. G.

Quote

Quote of 1, *divided by three times the Diam. of a Circle whose Periph. is* 1, *viz.* ,95493 —— } 1.047197

Product found as underneath. —— 864

Solid Content in Inches —— 904.778208

Quote of 1.047197 *divided by* 282 *is* —— .0037135

Product found as below. —— 864

Solid Content in A. G. —— 3.2084640

The segments Altitude 6 Inches subducted from the Axe 20 Inches leaves 14, to which add 10 the semi-Axe, the sum is 24, which Multip. by 36 the square of the Altitude, the Product is 864 as above.

23. *By the Diam. at Base and Altitude of any Segment: to find the Solidity.*

To the Area of the Diameter at Base, add four thirds of the Area corresp. to the Altitude, the sum multiplyed by Half the Altitude produces the Solid Content. Diamiter at Base 18. 33 Inches per Tab. of Areas of Circles —— .9358 A G.

Segment Altitude 6 Inches. 1003 whose $\frac{4}{3}$ is —— ,1337

Sum is —— 1.0695

Multiplyed by half the Altitude —— 3

Solid Content of the segment in A G. —— 3.2085

Applicable for finding the Content of the Crown of a Copper. But Crowns being generally more Parabolical then Spherical, Multiply the Area Corresp. to the segments Diameter by half the Altitude, and the Product is the Content. vid. Prop. XV.

VIII. Prop, To find the Content of the Regular Polygons.

Regular Polygons taken strictly, are such as have more sides, and Angles then four, and are all equal, as the *Pentagon*, *Hexagon*, *Heptagon*, &c.

1. *By the Side and Radius: to find the Area.*

Multiply the semi-perimeter, or Half sum of all the sides, by the Radius, or Pervend. falling from the Center upon the middle of any one of the sides, the Product is the Area in Inches, which Multiplyed or divided by the proper Multiplyer or Divisor taken out of the Tablet in P. 35 gives the Content in A G.

Now to find the Center, and Perpend. do thus; If the sides of the Figure are Even, draw a Right Line from any Angle to the opposite Angle, and cut it again, the intersection of those two Lines is the Center.

But if the sides are odd draw Lines from the middle of any two sides being odd to the opposite Angles, the intersection of those Lines shews the Center.

§. *Or by the side; to find the Perpend. Arithmetically.*

Multiply or divide the side of the given Polygon, by the proper Multiplyer or Divisor the Product, or Quote is the *Radius* of the Circle Inscribed.

Perpend. of a Trigon whose side is 1 *in Inches*	.288675
Given side in Inches	60
Perpend. (*rejecting the Cyphers*)	17.3205
Side 60 Multiply by 3, is 180, whose half is	90
Area in Inches	1558.8450
Multiplyer for A. G.	.0035461
Content at 1 Inch deep in A.G.	5.527820254 50
Quote of .288675 *divided by* 282,	.0010236 7
Side given in Inches	60
A. G. = Perp of a Trigon whose side is 60 Inches	.061410 10
Sem-perimeter, or half sum of the sides	90
Area in A. G.	5.527818 00

Polygons	Mult. for Inc.	Div. for Inc.	Mult. for A.G.	Div. for A.G.
Trigon	.288675	3.464103	.00102367	976. 877
Pentagon	.688192	1.453083	.00244040	409. 769
Hexagon	.866025	1.154701	.00307101	325. 626
Heptagon	1.038260	.963149	.00368177	271. 608
Octagon	1.207107	.828427	.00428052	233. 616
Enneagon	1.373739	.727940	.00487142	205. 279
Decagon	1.538819	.649841	.00545688	183. 255
Hendecag	1.702847	.587252	.00603847	165. 605
Dodecag	1.866028	.535808	.0066172	151. 123

Multiply

§. *By the Table for Gauging Right-Lined Figures.*

Multiply the Tabular Number correſp. to the Sem-perimeter or Radius by half the Contrary Number, the product is the Content at 1 Inch deep in A. G.

17.32 Inches the Radius per Tab —— ——.06142
Sem-perimeter, or half ſum of the ſides —— ——90
Content at 1 Inch deep in A.G. —— —— ——5.52780

§. *By the Table of Areas of Squares.*

A Geometrical mean between the *Perimeter, and Half the Radius or Perp; Or Sem-Perimeter, and Radius, is the ſide of a Square equal to the Area of the Pentagon;* with which enter the Table and take out the Correſp. Area.

17.32 Perpend. Multiplyed by 90 = 1558.8. Sq. R. is 39.48. per Tab. 5.5272 A.G.

2. *By the ſide; to find the Area without finding the Radius.*

Square the ſide, and Multiply or Divide it by the Tabular Number proper to the given Figure, the Product or Quote is the Area.

Area of a Trigon whoſe ſide is 1 in Inches —— ——.433013
60 Inches the given ſide ſquared —— —— ——3600
Area in Inches —— —— ——1558.846800
Quote of .433013 divided by 282, Or the Area of a Trigon whoſe ſide is 1. in A.G. —— ——.0015355
60 Inches the given ſide ſquared —— —— 3600
Content upon 1 Inch in A.G. —— ——5.5278000

Polygons	Mult. for Inc.	Div. for Inc.	Mult. for A.G.	Div. for A.G
Cylinder	.785398	1.273239	.0027851	359. 054
Trigon	.433013	2.309401	.0015355	651. 250
Tetragon	1.000000	1.000000	.0035461	282. 000
Pentagon	1.720478	.581233	.0061010	163. 908
Hexagon	2.598076	.384900	.0092130	108. 542
Heptagon	3.633910	.275185	.0128863	77. 602
Octagon	4.828427	.207107	.0171221	58. 404
Enneagon	6.181827	.161762	.0219215	45. 617
Decagon	7.694197	.129966	.0272844	36. 650
Hendecag	9.365659	.106773	.0332155	30. 110
Dodecag	11.196165	.089316	.0397030	25. 187

Note

Note that the *square of the Semi-side* Multiplyed by four times the Multiplicator or divided by one fourth part of the *Divisor* in this Tablet proper to the given Figure, and tenour of the Question produces or Quotes the *Area*.

The preceding Example I suppose is sufficient to illustrate the manner of working for all the Ordinate Polygons the operation being one and the same only you must be very careful in taking the Multiplyer or Divisor proper to the given Figure and intent of the Question.

2. *By the Area to find the Side.*

Multiply or Divide the Area by the Tabular Number of the contrary denomination for that Polygon, the square Root of the Product, or Quote is the side.

Multiplyer proper to the Trigon ——	—— 651. 25
Area given —— ——	—— 5.5278
Square Root 60 Inches is the side ——	—— 3599.979750

3. *By the Side; to find the Diameter.*

Divide the *Area*, by the *Semi-Perimeter*, or halfe sum of the sides, the *Quote is the Radius*, which doubled is the *Diameter*.

Semi-perim. 90) 1558. 8450 (17. 3205 = Radius

the Diameter is —— 34. 6410

IX. Prop. To find the Content of a Prism.

A *Prism* is a solid having two Bases equal, parallel, and alike situate; also in the *Peripetasma*, a right line may be every where applyed from Base to Base.

Under this Definition is comprehended a *Cylinder*, which is indeed a *Round Prism*. Multip. the Area of one of its Bases (*whether Triangular, Square, Oblong, Round Elliptical, Pentagonal &c.*) by the Altitude or Depth, the product is the solid Content.

Quote of .785398 divided by 282 is ——	,002785 1
Diam. 60 Inches Squared ——	—— 3600
Content at 1 Inch deep in A. G. ——	10. 026 3600
Depth in Inches —— ——	— —— 20
Whole Content in A. G. ——	200. 527 2000

Being Triple the Content of a Pyramid of the same Base, & Alti.

§. *Or By the Table for Gauging Circular Figures.*

Find

Find the Diam. in the Tab. & against it stands a Number, which Multip. by the Diam produces the Content at 1 Inch deep in A. G
Against 60 Inches Diameter per Table stands. —— .16711
Multiplyed by Inches —— —— 60
Content upon 1 Inch in A. G. —— 10.02660

Note, that the Table is made by a continual Addition.

And if the Inches proposed are more then are expressed in the Table then take the Number against Half, and Double it, the Sum is the Number sought.

§. *Or By the Table of Areas of Circles.*

Enter the Table with the given Diameter, and against it stands the Content. 60 Inc. Diam. per Tab. gives —— 10.0264 A.G.

Applicable for finding the Content of a Back in either of these Forms; As also a Tun the Bases, & sides above, & Below equal.

X. Prop. To find the Content of an Ellipsis, and Elliptical Solid.

1. *By the Diam. and Depth; to find the Content.*

As 14, to 11 Or 284, to 223; Or 452; to 355;
Or 1, to .785398; so is the Rectangle of the Transv. and Conjug. Diameters to the Area in Inches.
Or the Rectangle Divid. by 1.273239, Quotes the Area in Inc.
Or the Rectangle Multiplyed by, .0027851, or Divided by 359.0536 Quotes the Content at 1 Inch deep in A G. and that Multip. by the Depth produces the whole Content in A G.
Transverse Diam. 60 Multiplyed by 32 Conjug. Diam. = 1920
Proper Multiplyer —— —— —— .0027 1
Content in 1 Inch deep in A. G —— —— 5.34739-0
Depth in Inches —— —— —— 20
Whole Content in A. G. —— —— 106.9478400

§. *Or by the Table for Gauging Circular Figures.*

Enter the Table with either Diameter and against it stands a Number, which Multiplyed by the other produces the Content at 1 Inch deep in A. G.
Against 32 the Conjugate or shortest Diam. stands .08912
Multiplyed by the Transverse or Longest Diameter —— —— 60
Content upon 1 Inch in A. G. —— —— 5.34720

§. *Or By the Table of Areas of Circles.* Multip

Multiply the Transverse by the Conjugate Diameter and extract the square Root which is the Diameter of a Circle whose Area is equal to the Area of the Ellipsis, with which enter the Table, and against it stands the Content upon 1 Inch in A. G. *Note that by the Table of Squares you have the Square Root by inspection.*

60 Mult. by 32 = 1920 square Root is 43·82 Inches 5.7479 A. G

Applicable for finding the Content of a Back in Form of an Elipsis; As also an Eliptical Tun, the Bases above & below equal.

2. *By any two Diameters to describe an Elipsis.*

Draw two Lines equal to the given *Diameters* crossing one another at *right Angles*, and dividing each other into two equal Parts; then take between the Compasses half the *Transverse Diameter*, and setting one Foot in the Extream point of the *Conjugate Diameter*, cross the *Transverse Diameter* with an *Arch* at each end: then with the same extent set one Foot in the other extream point of the *Conjugate Diameter* and cross the two former *Arches*.

In those Points drive two Tacks or Pins; stick also another Tack in one of the extream *Points* of the *Conjugate Diameter*, then take a string and encompass these three Tacks, in form of a Triangle; pluck the string tite, and tye the two ends together and take out the Tack in the extream point.

Lastly, if you hold a Pencill close to the inside of the string, and carry it about the Tacks (*holding the string alwayes streight*) you will describe the *Elipsis or Oval*.

XI. Prop. To find the Content of a Pyramid.

A *Pyramid* is a solid having one right lined Base (*either Triangular, Round, Square, Oblong &c.*) from whence it decreases equally less and less till it end in a Point at the *Vertex*; Also in the *Peripetasma* a right Line may be every where applyed from the Base to the *Vertex*.

Under this Difinition is comprehended a *Cone* which is indeed a *Round Pyramid*.

3. By

1. *By the Area of the Base and Altitude: to find the Solid Content.*

Multiply the whole of the One, by one third part of the other, the product is the Content.

Base of a square Pyramid 144 Inches per Tab. —— 73.5319
Altitude 240 Inches whose One third is —— 80

Pyramids Content in A. G. —— 5882.5520
Diameter of a Cone's Base 144 Inches per Tab. —— 57.7518
One third of 240 the Altitude —— 80

Cone's Content in A. G. —— 4620.1440

Being One third of a Prism of the same Base, and Altitude.

2. *By the Altitude, and Base; to find the Hypothenuse.*

To the sq. of of the Altitude, add the sq. of the Sem-Base, the sq. R. of the sum is the Hypothenuse or slant Height.

Altitude 240 Inches squared —— 57600
Sem-Base 72 Inches squared —— 5184

Square Root 250.568 is the Hypoth. —— 62784

3. *By the Hypothenuse and Base; to find the Altitude.*

From the sq. of the Hypoth. Subduct the sq. of the Sem-Base, the sq. R. of the Remainder is the Altitude.

Hypoth. 250.568 sq. —— 62784
Sem-Base 72 sq. —— 5184

Sq. R. 240 is the Altitude —— 57600

4. *By the Hypothenuse and Altitude to find the Base*

From the Square of the Hypothenuse or slant Height, Subduct the square of the Altitude; the square Root of the Remainder is the Sem-Base.

Hypoth. 250.568. sq. —— 62784
Altitude 240 —— sq. —— 57600

Sq. R. —72 is the sem-Base —— 5184

XII. Prop. To find the Content of the Frustum of a Pyramid or Cone.

The *Frustum of a Pyramid* is a solid whose Bases are in the Form of any Regular Polygon, parallel, alike, and alike Situate; and also a right Line may be every where applyed in the *Perpet asma* from Base to Base.

1. *By the Dimentions to find the Content in A. G.*

As the Difference between the Sides, or Diameters 36. to the Frustum's Altitude 60 Inches. So is the greater Side, or Diameter 144 :: to the whole Altitude 240 Inches.
From which Subduct the Frustum's Altitude — 60
Rests the Altit. of Top part of lesser Pyramid. — 180
Then from the solid Content of the whole Pyramid, subduct the Content of the Lesser Pyramid, the Remainder is the Content of the Frustum.
Whole Sq. Pyramids Content found by Prop. XI. — 5882.5520
108 lesser Base = 41.3617 Multip. by 60. one third of the Altitude of the Lesser Pyramid. — 2481.7020
Content of the Frustum. — 3400.8500
Content of the whole Cone found by Prop. XI. — 4620.1440
108 Base = 32.4854 Multip. by one third of Alt. — 1949.1240
Content of the Frustum — 2671.0200

§. *Or you may find the Content without obtaining the whole Altitude*

From the square of the sum of the Sides or Diameters Above, and Below, Subduct the Rectangle of the Sides, or Diameters the Remainder Multiplyed or Divided by the Multiplyer or Divisor proper to the given Figure taken out of the Tablet in P. 43, gives the Area in Inches or A. G. and that Multiplyed by the Altitude produces the Content of the Frustum in A. G.

Sides

Sides or Diam. { 100 / 78 } Depth 22 Inches.

Sum- 178 sq. —— 31684
100 Multip. by 78 —— 7800
proper Divisor —846) 23844 (28.2317 A. G.
Depth in Inches —— —— 22
Content. of the Frustum of a Sq. Pyramid. in A.G. 621.0974
1077.161) 23844.000(22.1731 A.G.
Depth in Inches —— —— 22
Content of the Frustum of a Cone in A. G. 487.8082

Note that these Rules are general for the Frustum of any of the Ordinate Polygonial Pyramidal Solids.

James Edge

Polygons	Mult. for Inc	Div. for Inc	Mult. for A.G	Div. for A.G
Cone	.261799	3.819717	.00092837	1077. 161
Trigon.	.144338	6.928203	.00051184	1953. 750
Tetragon.	.333333	3.000000	.00118203	846. 000
Pentagon	.573493	1.743699	.00203366	491. 724
Hexagon	.866025	1.154701	.00307101	325. 626
Heptagon	1.211303	0.825555	.00429543	232. 806
Octagon	1.609476	0.621320	.00570737	175. 212
Eneagon	2.060609	0.485286	.00730717	136. 851
Decagon	2.564732	0.389898	.00909481	109. 950
Hendecag.	3.121886	0.320319	.01107183	90. 330
Dodecag.	3.732055	0 267948	.01323422	75. 561

Note that the Multip. in this Tablet, are One third of the Multip. and the Divis. Triple the Divis. in the Tablet. p. 37.

§ *Or by the Tables for Gauging Right-Line, or Circular Figures.*

Enter the proper Table with either Base, and take out the Number corresp. which Multip. by the other Base reserving the Product: then take out the Number against the Differ. of Dimentions, and Multip. it by One third part of the said Differ. Or Multip. One Third part of the Number against the Differ. of Dimentions by the Differ.

Add the Product to the reserved Product, and the Sum is the Content at 1 Inch deep in A. G. which Multip. by the Depth produces the whole Content. 78

78 Lesser Base per Tab. for Gauging Right-lined Fig. is. 27660
——— Multip. by the Greater Base ——— 100
——— Product Reserve ——— 27.66000
22 Dif. of sides = 07801 the $\frac{1}{3}$ .02600 Multip by 22 = 57200
Content at 1 Inch deep in A. G. ——— 28.23200
Depth in Inches ——— 22
Cont. of the Frustum of a sq. Pyr, in A. G. ——— 621.10400
78 Lesser Base per Tab. for Gauging Circ. Figures is — .21724
Multip. by the Greater Base. ——— 100
Product is ——— 21.72400
22 Dif. Diam. = .06127 the $\frac{1}{3}$ is .02042 Mult. by 22 = .44924
Content at 1 Inch deep in A. G. ——— 22 17324
Depth in Inches ——— 22
Cont. of the Frustum of a Cone in A. G. = ——— 487.81128

§. *Or by the Tables of Areas of Squares, or Circles.*

The Sem-differ. of Dimentions added to the Lesser, or Subducted from the Greater gives an Arithmetical mean, with which enter the proper Table and take out the Area Corresp. to which add One Third part of the Area against the Sem-dif. of Dimentions the Sum is the Content upon 1 Inch in A. G. which Multip. by any Depth produces the Whole Content in A. G.

89 Arith. mean per Tab. of Areas of Squares give 28.0887 A. G.
11 Sem-dif. of Sides = 4291 whose $\frac{1}{3}$ is ——— 1430
Content at 1 Inch deep in A. G. ——— 28.2317
Depth in Inches ——— 22
Frustum's Content in A. G. ——— 621.0974
89 Arith. mean per Tab. of Areas of Circles — 22.0608
11 Sem-dif. of Diam. = 3370 the $\frac{1}{3}$ is ——— 1123
Content upon 1 Inch in A. G. ——— 221731
Depth in Inches ——— 22
Frustum's Content in A. G. ——— 487.8082

§. Or

§. *Or thus By the Tables of Areas of Squares or Circles.*

Multip, the Dimenti ons together and extract the Sq. R. with which enter the proper Table and take out the corersp. Area, to which add one Third part of the Area against the Differ. of Dimentions the Sum is the Content at 1 Inch deep in A. G. which Multip. by the Depth, and you have the whole Content in A. G

100 Multip. by 78 = 7800 whose Sq. R. is 88.32 per T. — 27.6611
22 Differ. of Sides = 1.7163 the $\frac{1}{3}$ is — .5711

Content at 1. Inch deep in A. G. — 28.2322
Depth in Inches — 22

Whole Content in A. G. — 621.1084
88.32 per Table of Areas of Circles — 22.7249
22 Differs of Diam. = 1.3480 whose $\frac{1}{3}$ is — 4493

Content at 1 Inch deep in A. G — 22.1742
Depth in Inches — 22

Whole Content in A. G. — 487.8324

Applicable for finding the Content of a Tun the Bases Above, and Below unequal, but equal Sides at each Base.

Note, That by the 6. 7. 8. and Rules of Prop. 1. Chap. III. you may find the *Diagonal, Altitude, Hypothenuse, and Diameters of the Frustum of a Pyramid, or Cone.*

2. *By the Dimentions; to find the Content at every Foot, Half Foot, or Inch.*

Divide the *Differ. of Dimentions* Above and Below by the Altit. or Depth, the Quote is the Decrement, or Increment at 1 Inch distance in the Depth, commonly called the *Subducend, or Addend*: And at every Foot, Half-Foot, or Inch of the Gradation Note the Dimentions, and if these agree with the Dimentions Actually taken it is a Confirmation that the Tun is the Frustum of a Pyramid, or Cone, tho not one of a Thousand are so exact.

Then from the proper Table insert against each respective Dimention its respective Area. (*Or calculate them according to the precedent Rules*) which Multip. by 12 or 6 gives the Content of that Foot, or Half-Foot, And in either Case the total sum is the whole Content in A. G.

Now

Now to take the Dimentions Actually do thus

With a long ſliding Rule take the Dimentions both Above and Below croſſe-wiſe, and if they differ, *(provided the Difference do not exceed two Inches)* take an Arithm. Mean, then ſet the Rule to that Mean, and ſee what part of the Tun will bear it; *this done*, from the two ends of the Rule ſo ſet, with Chalk draw two Lines perpendicularly from the Top to the Bottom of the Tun, and with another ſmal Rule croſs with Chalk either of the Lines before drawn, *Six Inches, Three Inches, or Half an Inch from the Top*, according as you intend to Gauge the Tun, whether to the Foot, Half-Foot, or Inch; *having due regard to the perpendicular Altitude, and Fall*; Then take *Twelve Inches, Six Inches, or one Inch from that Croſs*, and make the like ſtroke, and ſo at every Foot, Half-Foot, or Inch of the perpendicular Depth make the like ſtrokes, do the ſame on the oppoſite Line, for by this meanes the Tun is reduced into ſo many Squares, or Cylinders: Then take the Dimention of each reſpective Segment by placeing the two ends of the Rule in thoſe croſs ſtrokes, entring them gradually in a Paper *(begining at Top the Dimentions being moſt commonly taken downwards)* And if any odd Inches in Depth remain take, the Dimention in the midle of thoſe odd Inches. Then finiſh the work as above directed.

If you take the Dimentions in the Side Line *(as is uſuall)* you muſt make an allowance accordingly.

3. *To find the Drip, or fall of any Tun.*

Pour in ſo much Liquor as will juſt cover the Bottom, Half the Depth taken in the deepeſt place is the *Fall*, which ſubducted from the whole Depth, leaves the true Depth for Inching that Tun.

Or if the Tun be quite full, then an Atithm. Mean between the Leaſt and greateſt Depth, is the Tru Depth.

Much conteſt there is about this Rule, but this I ſuppoſe ſufficient for ſatisfaction of the Country Gauger.

Exa.

Exx. 1. *There is a Tun which is the Frustum of a Square Pyramid whose sides Above are each* 100 *Inches Sides Below* 78 *Inches, and Depth* 22 *Inches.*

To find the Content at every half Foot in A. G.

Sides { Above 100 / Below 78

Depth 22) 22 (1 = Subducend.

Inches in a Quarter of a Foot — 3

Product — 3 which subducted from

Great Diam. at Top — 100

Diam. at 3 Inch. from Top — 97

Subd. for half a Foot — 6

Diam. at 9 Inches from Top — 91

Subducend — 6

Diam. at 15 Inches from Top — 85

Diam. at 20 Inch. from Top — 80

Note, that the Altitude of the last Segment being 4 Inches you must take the Mean in the Middle of it according to the former Direction. (*that is*) 5 Inches from the last Mean.

	Depth	M.	S.	Ar. 1 Inc. deep.	Ar. 6 Inc. deep.
	6	97	33. 3693	200. 1918	
	6	91	29. 3583	176. 1918	
I.S.T.22.	6	85	25. 6209	153. 7236	
	4	80	22. 6950	90. 7800	
	22			620. 8872	

To find the Content at every Inch in A. G.

Subducend found — 1

Halfe of an Inch — .5

Product — .5 Subd. from

Greater Diameter at Top. — 100

Diameter at Half an Inch from Top. — 99.5

Depth

	Depth	M.S.	Ar. 1 Inc deep	Dep.	M S.	Ar. 1 Inc. deep
	1	99.5	31. 1073	12	88.5	27. 7740
	2	98.5	34. 4051	13	87.5	27. 1498
	3	97.5	33. 7101	14	86.5	26. 5328
	4	96.5	33. 0222	15	85.5	25. 9225
	5	95.5	32. 3413	16	84.5	25. 3200
	6	94.5	31. 6676	17	83.5	24. 7243
1.S.T.22.	7	93.5	31. 0009	18	82.5	24. 1356
	8	92.5	30. 3413	19	81.5	23. 5541
	9	91.5	29. 6888	20	80.5	22. 9756
	10	90.5	29. 0434	21	79.5	22. 4122
	11	89.5	28. 4051	22	78.5	21. 8520
			348. 7331			272. 3573
						348. 7331
						621. 0904

Example 2. *There is a Tun which is the Frustum of a Cone, whose Diam. Above is* 100 *Inches* Diam. *Below* 78. *and Depth* 22. *Inches.*

To find the Content at every Half-foot in A. G.

Here the extream Diam. and Depth being the same as in Example 1. the rest of the Diam. must consequently be the same, so that if you Multiply the Area Coresp. to each Diam. by 6, and add the Products together, the Sum is the Whole Content in A. G.

	Depth	M. D.	Ar. 1. Inch deep.	Ar. 6. Inches deep
	6	97	26. 2050	157. 2200
	6	91	23. 0634	138. 3804
2R.T.22	6	85	20. 1223	120. 7338
	4	80	17. 8246	71. 2984
		22		487. 6426

To find the Content at every Inch in A. B.

	Depth	M.D.	Ar. 1	Inc. deep	Dep.	M.D.	Ar. 1	Inc. deep
	1	99.5	28.	5732	12	88.5	21.	8136
	2	98.5	27.	0217	13	87.5	21.	3234
	3	97.5	26.	4759	14	86.5	20.	8388
	4	96.5	25.	9355	15	85.5	20.	3598
	5	95.5	25.	4008	16	84.5	19.	8865
2. R. T. 22.	6	94.5	24.	8716	17	83.5	19.	4184
	7	93.5	24.	3480	18	82.5	18.	9564
	8	92.5	23.	8300	19	81.5	18.	4992
	9	91.5	23.	3176	20	80.5	18.	0481
	10	90.5	22.	8107	21	79.5	17.	6023
	11	89.5	22.	3093	22	78.5	17.	1625
			273.	8943			213.	9983
							273.	8943
							487.	8037

§. *Another way of finding the Content gradatim from Inch to Inch. according to M. D.*

1. One Third part of the Square of the Subducend, or Addend (*found as before directed*) is the *first Stationary Number*.

2. The Rectangle of the Greater, or Lesser Base, and Subducend, or Addend is the *Second Stat. Number*.

3. The Sq. of the Greater, or Lesser Base is the *Third Stat. Number*, *all of them being first severally Multip. or Divided by the Multip. or Divisor in Tab. p. 37. proper to the given Figure and purport of the Question.*

Then if you intend to Gauge it from the Greater Base.

Multp. the *First Stat. Numb.* by the Depth, and Subduct the Product from the *Second Stat. Numb.* Multip. the Remainder by the Depth the Product Subducted from the *Third Stat. Numb.* Leaves a *Mean Area*, which Multip. by the Depth produces the Whole Content.

But if you intend to Gauge it from the Lesser Base.

Multip. the *First Stat. Numb.* by the Depth, and to the Product add the *Second Stat. Numb.* Multip the Sum by the Depth, and to the Product add the *Third Stat. Numb.* the Sum is a *Mean-*

Area, which multip. by the Depth produces the whole Content.

Exa. 1. *Sides* { 100 / 78 } Depth 22 Inches

Depth 22) 22 (1 = *Subd. or Addend*

From the Great[er] *Base*

One 3d of the Sq. of 1, is 33333	Divid.	.001182 Fir.	Stat. Nu.
100 Gr. Ba. Mult. by 1. is — 100	by 282	.354610 Sec.	
100 Gr Base Sq. is — 10000	Quotes	35.46100 Th.	

First Stat. Numb.	.001182
Depth in Inches	22
Product Subducted from	.026004
Second Stat. Numb.	.354610
Remainder	.328606
Depth	22
Product Subd.	7.229332
Third Stat. Number	35.46100
Mean Area in A. G.	28. 231668
Depth	22
Whole Content in A. G.	621.096696

2 *From the Lesser Base.*

First Stat. Number	.001182
Depth	22
Product	.026004
Second Stat. Number add	.276596
Sum	.302600
Depth	22
Product	6.657200
Third Stat. Number add	21.57447
Mean Area in A. G.	28.231670
Depth	22
Whole Content in A. G.	621.096740

Now these last Stat. Numbers are thus found.

One 3d of the Sq. of Ad. 33333	Mul. by 005461	.001182 — 1	
Less Base Mul. by Adden. — 78	or Divi. by 282	.276599 — 2	
Less Base 78 Sq. is — 6084		21.57447 — 3	

Exa[r]

Exa. 2. Diam. { 100 / 78 } Depth 22 Inches

Depth—22) 22 (1 = *Subd. or Addend.*

1. *From the Greatest Base.*

One third of 1 Sq. 1—33333	M. by. .0027851	.0009284	—1
100 Gr. Ba. Mult. by 1.—100	or Divided by	.2785100	—2
100 Gr. Base Sq. —10000	359.054	27.85 000	—3

First Stat. Numb.	.0009284
Depth	22
Product Subduct from	.0204248
Second Stat. Numb.	.27851
Remainder	.2580852
Depth	22
Product subd.	5.6778744
Third Stat. Number	27.851000
Area of a Mean Circle in A. G.	22.1731256
Depth	22
Whole Content in A. G.	487.8087632

2. *From the Lesser Base.*

One 3d of 1 Sq.—33333	M. by. .0027851	.0009284	—1
78 Less Base Mult. by 1.78	or Divided by	.2172378	—2
78 Less Base Sq. —6084	359.054	16.9445484	—3

First Stat. Numb.	00.09284
Depth	22
Product	.0204248
Second Stat. Number add	.2172378
Sum	.2376626
Depth	22
Product	5.2285772
Third Stat. Number add	26.9445484
Area of a Mean Circle	22.1731256
Depth	22
Whole Content in A. G.	487.8087642

§. *A New and Compendious way of Gauging, and Inching the Frustum of a Pyramid or Cone, according to the Method used by Mr Christopher Banson late Generall Rider for York-shire &c.*

Rule

Rule, One sixth part of the Area Correspondent to the Difference of Sides, or Diameters (*taken out of the Proper Table, or Calculated as already directed*) subducted from the Area of the Side or Diam. at Top, leaves a Number (*which Reserve.*)

Then Divide the Differ. of the Areas of Top and Bottom by the Depth, the Quote is the common *Subducend*, or *Addend*, (*according to the Position of the Greater or Lesser Base.*)

If the Greater Base be at Top. Subduct *half the Subducend* from the *Reserved Number*, the Remainder is the Area of the first Inch, then by a continual Subduction of the *whole Subducend*, you have the Area of each respective Inch to the Bottom.

But if the Lesser Base be at Top, Add *Half the Addend* to the *Reserved Number* the Sum is the Area of the first Inch, then by a continual Addition of the *whole Addend* you have the Area of each respective Inch to the Bottom.

Sides { Above 100 — 35.4610 } A. G.
{ Below 78 — 11.5745 }

Depth 22) 13.8865 (.6312 — Subd.
.3156 — Half.

Side at Top 100 Inches per Table of Sq. — 35.4610 A.G.
22 Dif. of Sides — 1.7163 whose one sixth part is — .2860
Rest Reserved Number — 35.1750
Half the Sub. Subd — .3156
Area at 1 Inch from Top — 34.8594
Whole Subducend — .6312
Area at 2 Inches from Top — 34.2282
Whole Subd. — .6312
Area at 3 Inches from Top — 33.5970

Diam. { Above 100 — 27.8510 } A.G.
{ Below 78 — 16.9445 }

Depth. 22) 10.9065 (.4957 Subd.
.2478 Half

Diam. at Top 100 Inches per Tab. of Circles — 27.8510
22 Dif. of Diam. — 1.3480 the one Sixth is — .2247 Subd.
Remainder Reserve — 27.6263
Half the Subducend — .2478 Subd.
Area at 1 Inch from Top — 27.3785
Whole Subd. — .4957

Area at 2 Inches from Top ——— 26.8828
Whole Subd ——— 4957
Area at 3 Inches from the Top ——— 26.3871

	Depth	Ar.	In. dp.
	1	34.	8594
	2	34.	2282
	3	33.	5970
	4	32.	9658
	5	32.	3346
	6	31.	7034
	7	31.	0722
	8	30.	4410
	9	29.	8098
1. S. T. 22	10	29.	1786
	11	28	5474
	12	27	9162
	13	27	2850
	14	26	6538
	15	26	0226
	16	25.	3914
	17	24.	7602
	18	24.	1290
	19	23.	4978
	20	22.	8666
	21	22.	2354
	22	21.	6042
		621.	0996

	De	Ar.	In. dp.
	1	27.	3985
	2	26.	8828
	3	26.	3871
	4	25.	8914
	5	25.	3957
	6	24.	9000
	7	24.	4043
	8	23.	9086
	9	23.	4129
2. R. T. 22.	10	22.	9172
	11	22.	4215
	12	21.	9258
	13	21.	4301
	14	20.	9344
	15	20.	4387
	16	19.	9430
	17	19.	4473
	18	18.	9516
	19	18.	4559
	20	17.	9602
	21	17.	4645
	22	16.	9688
		487.	8183

XII. Prop. To find the Content of a Prismoid.

A Prismoid. is a Solid contained under six plain Surfaces, whereof the Two Bases are to be Rectangular Parallelograms, Parallel one to another, and alike Situate, *(that is)* So Situate that the Rectangular Conjugates in both Bases may be inserted by two and the same Planes, and the remaining four Planes are the *Peripetasma.*

But if the *Peripetasma* be not constituted by four flat sides, but

it shall be cloathed about with Curvature from Circles, or Ellipsis; the Solid is then called a *Cylindroid*.

1. *By the Dimentions to find the Content in A.G.*

To the Length, or Transv. Diam. Above, add Half the Length, or Transv. Diam. Below, and Multip. the Sum by the Bredth, or Conjug. Diam. Below.

To the Length or Transv. Diam. Below, add half the Length or Transv. Diam. Above and Multip. the sum by the Bredth, or Conjug. Diam. Above.

The Sum of these two Products Multip. or Divid. by the proper Multip. or Divisor taken out of the Tablet P. 44. gives the Area in Inches or A. G. which Multiy. by any Depth produces the Content at that Altitude.

Length Above—100	Length Below—90
Half Len. Below 45	Half Len. Above-50
Sum ——— 145	Sum ——— 140
Bred. Above—90	Bred, Below—80
1. Product—13050	2. Product—11200
2. Product—11200	

846) 24250 (28,6643 A. G.

Depth in Inches ——— 20

Prismoids Cont. in A G. — 573,2860

1077,161) 24250.000 (22,5138 A. G.

Depth in Inches ——— 20

Cylindroids Cont. in A.G. 450.2760

2. *Or by the Tables of Areas of Squares.*

Find a Geometrical mean between the Length, and Bredth, or the Transv. and Conjug. Diam. at Top, as also at Bottom (*which you may readily do by the Table of Squares*) So shall you have the Two ends of the Frustum reduced either to a Square, or a Circle, then finish the Work as directed Prop XII. for the Frustum of a Pyramid or Cone.

Dim. { Above 190 by 90 = 9000 Sq. R. ——— 94,87
{ Below 90 by 80 = 7200 Sq. R ——— 84,85

Sum ——— 179,72

89. 86 Half Sum per Tab. of Areas of Squares.-28.6341 A.G.

5 Sem-dif. of Sq. = 0885 whose one third is ——— ,0295

Depth

Content upon 1 Inch in A. G. ——— 28,6636
Depth in Inches ——— 20
Prismoid's Content in A. G. ——— 573,2720
89,86 Half sum per Tab. of Areas of Circles — 22,4892 A. G
5 Sem-dif. of Diam. ,0696 the One third is —. 0232
Content upon 1 Inch in A. G. ——— 22,5124
Depth in Inches ——— 20
Cylindroids Content. in A. G. ——— 450,2480

Applicable for finding the Content of a Tun whose Bases a Sides Above, and Below are unequal.

2 *By the Dimentions to find the Content at every Foot, half Foot, or Inch.*

Divide the difference of the Lengths, or Transv. Dia Above and Below by the Depth, the Quote is the Decreme or Increment at 1 Inch distance in the Depth which call and reserve it for a common *Subducend*, or *Addend*, accordi to the Position of the Greater or Lesser Base.

Divide also the Difference of the Bredths or Conjug Diam. Above and Below by the Depth and the Quote cal and reserve it for a *Subducend*, or *Addend*, according to the P sition of the Bases; And at every Foot, half Foot, or Inch the Gradation Note the Dimentions, Then find the seve Geometrical Means, and against them insert their respect Areas, which Multip. by 12. or 6 Inches gives the Cont of that Foot, or half Foot, and in all the three Cases the S is the whole Content in A. G.

Now to take the Dimentions actually do thus. ———

With a slideing Rule take the Dimentions crosswise (*t is*) haveing crossed it one way at the greatest you can g with chalk make two short lines peprendicular wise, t placeing the ends of the Rule first in one, and then in other of those lines set off a convement distance what it wa of closing divide in the midst, and from thence draw L on both sides from the Top to the Bottom of the Tun, and those Lines place the Inches, as directed Prop. XII. accordi as you intend to Gauge the Tun, whether to the Foot, h Foot, or Inch, taking the Dimentions of each respective S

ment and inserting them in a Paper calling them the Lengths, and Breaths, or Transv. and Conjugate Diameters.

If any odd Inches remain, take the Dimentions in the middle of those odd Inches, and finish the operation as above directed.

I have known some Practical Gaugers use an Arithmetical Mean, which may serve in small Vessels where the extream Dimentions differ but a little, but in large Tuns I would by no means advise it.

If you take the Dimentions in the Side line you must allow accordingly.

Exa. I. There is an Oblong Tun whose Length Above is 100 *Inches, and Bredth* 90, *the Length Below* 90 *Inches, and Bredth* 80, *and depth* 20 *Inches.*

1. *To find the Content at every half Foot in A. G.*

Length { Above 100 / Below 90 } Inches

Depth 20) 10.0 (.5 Subd
One fourth of a Foot — 3
1,55
Greater Length — 100
Len. 3 Inc. from top 98,5
Subd. for half a Foot 3
Len. 9 Inc. from top 95,5
2
Le. 15 Inc. from top 92,5
2
Le. 19 Inc. from top 90,5

Bredth { Above 90 / Below 80 } Inches

Depth 20) 10.0 (.5 Subd.
One fourth of a Foot — 3
1,55
Greater Bredth — 90
Bredth 3 Inches — 88,5
3
Bredth 3 Inches — 85,5
3
Bredth 15 Inches 92,5
2
Bredth 19 Inches — 80,5

	Dep.	Leng.	Brea.	Geo. M.	Ar. 3 Inc. deep	Ar. 6 Inc. deep
2 Ob. T. 20	6	98	88,5	73. 31	30. 9015	285. 4090
	6	95	85.5	90. 35	28. 9472	273. 6832
	6	92	82.5	87. 35	27. 0563	262. 3376
	2	90	80.	85. 33	25. 8326	251. 6640
	20					1573. 6940

2 To

2. *To find the Content at every Inch in A. G.*

Subducend found ——— ,25 Subducend found ——— ,25
Half of an Inch ——— ,25 Half of an Inch ——— ,25
325 325
Greater Length ——— 100 Greater Bredth ——— 90.
Half an Inch from Top -99,75 Half an Inch from Top -89,75
Subducend for 1 Inch ——— ,25 Subducend for 1 Inch ——— ,325
Len. 1.5 Inches from top 99,25 Bre. 1.5 Inches from top 89,25

	Depth	Length	Bredt.	G.M.	Ar. 1 Inch de
	1	99.75	89.75	94. 6	31. 7349
	2	99.25	89.25	94. 1	31. 4000
	3	98.75	88.75	93. 6	31. 0672
	4	98.25	88.25	93. 1	30. 7362
	5	97.75	87.75	92. 6	30. 4070
	6	97.25	87.25	92. 1	30. 0795
	7	96.75	86.75	91. 6	29. 7538
	8	96.25	86.25	91. 1	29. 4298
	9	95.75	85.75	90. 6	29. 1077
	10	95.25	85.25	90. 1	28. 7873
1.Ob.T.20	11	94.75	84.75	89. 6	28. 4687
	12	94.25	84.25	89. 1	28. 1518
	13	93.75	83.75	88. 6	27. 8367
	14	93.25	83.25	88. 1	27. 5234
	15	92.75	82.75	87. 6	27. 2119
	16	92.25	82.25	87. 1	26. 9022
	17	91.75	81.75	86. 6	26. 5942
	18	91.25	81.25	86. 1	26. 2880
	19	90.75	80.75	85. 6	25. 9839
	20	90.25	80.25	85. 1	25. 6809
					173. 1448

Exa. 2. There is an Elliptical Tun whose Transv. Diam. Above is 100 Inches and Conjug. Diam. 90; the Transv. Diam. Below 90 Inches and Conjug. Diam. 80. And the Depth 20 Inches.

1. *To find the Content at every Half Foot in A. G.*

Here the Dimentions being the same as in *Exa.* 1. the rest of the Dimentions must also be the same, so that if you Multip. the Area corresp. to each Diam. by 6 and add the Products together the sum is the whole Content in A. G.

	Depth	T.D.	C.D.	G. M.	Ar. 1 Inc. deep.		Ar. 6 In. deep.	
2. Ell. T. 20	6	98.5	88.5	93.35	24.	2700	145.	6200
	6	95.5	85.5	90.35	22.	7351	136.	4106
	6	92.5	82.5	87.35	21.	2501	127.	5024
	2	90.5	80.5	85.25	20.	2884	40.	5768
	20						450.	1098

2. *To find the Content at every Inch in A. G.*

	Depth	Length	Bredth	Geo. Mea		Ar. 1. Inch deep.	
2. Ell. T. 20	1	99.75	89.75	94.	6	24.	9243
	2	99.25	89.25	94.	1	24.	6615
	3	98.75	88.75	93.	6	24.	4001
	4	98.25	88.25	93.	1	24.	1402
	5	97.75	87.75	92.	6	23.	8816
	6	97.25	87.25	92.	1	23.	6244
	7	96.75	86.75	91.	6	23.	3685
	8	96.25	86.25	91.	1	23.	1141
	9	95.25	85.75	90.	6	22.	8611
	10	95.15	85.25	90.	1	22.	6095
	11	94.75	84.75	89.	6	22.	3592
	12	94.25	84.25	89.	1	22.	1104
	13	93.75	83.75	88.	6	21.	8629
	14	93.25	83.25	88.	1	21.	6169
	15	92.75	82.75	87.	6	21.	3722
	16	92.25	82.25	87.	1	21.	1289
	17	91.75	81.75	86.	6	20.	8875
	18	91.25	81.25	86.	1	20.	6465
	19	90.75	80.75	85.	6	20.	4074
	20	90.25	80.25	85.	1	20.	1697
						450.	1464

§ *Another*

§ *Another Way of finding the Content gradually from Inch to Inch.*

Exa. 1. Length { Above 100 / Below 90 } Inches Breadth { Above 90 / Below 80 }

Dep.—10) 10 (,5 Dep.—10) 10 (,5

1. *From the Greater Base.*

1. One third part of the Rectangle of the two Subducends found as before directed is the first Stationary Number.

2. The Sem-sum of the Rect-angle of the two Greater Dimentions Multip. by their Alternate Subducends is the second Stationary Number.

3. The Rectangle of the Greater Base is the Third Stationary Number. all of them being first severally Multip. or Divided by the Multip. or Divisor in Tab. p. 37. proper to the given Figure, and intent of the Question.

D—,5	Grea Len.-100	Gr.Bred.90	Gr.Len.100
d—,5	d.——,5	D.——,5	Gr.Br. 90
Rectan. ,25	50,0	45,0	Rect.An.9000
	45,0		
the ⅓ is ,0833333	Sum——95.0		
	Half——47,5		

Then Multip. the first Stat. Numb. by the Depth, and Subd. the Product from the second Stat. Numb. Multip. the Remainder by the Depth, the Prod. Subduc. from the third Stat. Numb. leaves a mean Area which Multip. by the Depth produces the whole Content.

,0833333 } Multip. by ,003546 1 or { ,000295=1 } Stat.
47,5 } Divided by 282 gives { ,16844=2 } Numb.
9000 { 31,9149=3 }

First Stat. Numb. —— ,000295
Depth —— 20
Prod. Subd. —— ,005910
Second Stat. Number —— ,16844
Remainder —— ,1625340
Depth —— 20

Prod. Subd. ——— 3,2506000
Third Stat. Number ——— 31,9149
Mean Area in A.G. ——— 28,6643000
Depth ——— 20
Whole Content in A. G. ——— 573,2860000

2. *From the Lesser Base.*

1. One Third part of the Rect-Angle of the two Addends is the First Stationary Number.

2. The Semi-sum of the Rectangle of the two Lesser Dimentions Multip. by their Alternate Addends is the second Stationary Number.

3. The Rectangle of the Lesser Base is the Third Stationary Number. All of them being first severally Multip. or Divided by the Multip. or Divisor in Tab. p. 37. proper to the given Figure and intent of the Question.

Then Multip. the First Stat. Numb. by the Depth, and to the Product add the second Stat. Numb. Multip. the sum by the Depth, and to the Product add the third Stat. Numb. the sum is a mean Area which Multip. by the Depth produces the whole Content.

D—,5	Less. Len 90	Less. Bred. 80	Gr. Br.—90
d—,5	d —— ,5	D ——— ,5	Less. Br—80
Rectan. ,25	45,0	40,0	Rectan. 7200
the $\frac{1}{3}$ is ,0833333	40		
	Sum-85,0		
	Half-42,5		

,0833333 } Mult. by ,0035461 { ,0002955 = 1 }
42,5 } or divided by 282 { ,150709 = 2 } Stat. Numb.
7200 } gives { 25,53192 = 3 }

First Stat. Number ——— ,0002955
Depth ——— 20
Product Add ——— ,0059100
Second Stat. Number ——— ,150709
Sum ——— ,156619
Depth ——— 20
Product. Add ——— 3,132380

Third

Third Stat. Number ——— 25, 53192
Mean Area in A. G. ——— 28. 6643000
Dpth in Inches ——— 20
Whole Content in A. G. ——— 573, 2860000

Exa. 2. Diam. { Above 100 / Below 90 } Inc. Conj. Diam. { Above 90 / Below 80 }

Depth. 20.) 10 (,5 Depth—20)10 (,5

1. *From the Greater Base.*

,00833333 / 47,5 / 9000 } Multip. by ,0027851 or Divided by 359, 054 gives { ,00023209 = 1 / ,13229225 = 2 / 25,0659000 = 3 } Stat. Num.

First Stat. Number ——— ,00023209
Depth ——— 20
Product Subd. ——— ,00464180
Second Stat. Numb. ——— ,13229225
Remainder ——— ,12765045
Depth ——— 20
Product Subducted ——— 2.55300900
Third Stat. Number ——— 25,0659000
Area of a Mean Circle in A. G. ——— 22.51289100
Depth in Inches ——— 20
Whole Content in A. G. ——— 450,25782000

2 *From the Lesser Base*

,00833333 / 42,5 / 7200 } Multip. by ,003561 or Divid. by 359,054 giv. { ,00023209 = 1 / ,11836675 = 2 / 20,0527200 = 3 } Stat. Num.

First Stat. Numb. ——— ,00023209
Depth ——— 20
Product Add ——— ,00464180
Second Stat. Number ——— ,11836675
Sum ——— ,12300855
Depth ——— 20
Product add ——— 2,46017100
Third Stat. Number ——— 20,0527200
Area of a Mean Circle in A. G. ——— 22,51289100
Depth in Inches ——— 20
Whole Content in A. G. ——— 450,25782000

Note, that you may Inch them from either Base by finding the first Frustum, and First, Second, and Third Differences as directed in my Guide for the Practical Gauger.

XIV. Prop. To find the Content of a Spheroid.

A Spheroid or Long Sphere is a Solid Figure made by the Revolution of half an Ellipsis about its Transverse Diam. or Axe.

1. *By the Axe, and Lesser Diam; to find the Content.*

Mult. the Sq. of the Lesser Diam. by the Axe, the Product Multip. by, 523598 or divided by 1.909855 Produces the Content in Inches, and that Multip. or divided by the Multip. or Divisor taken out of the Tablet page, 37. gives the Content in A. G.

Or the Rectangle of the Greater, and Lesser Diam. Multip. by, 0018567, or Divided by 538, 58, gives the Content in A. G.

Axe or Greater Diam. ——— 60,044
Lesser Diam. 26. Sq. ——— 676
40589,744
Content of a Sphere whose Axe is 1 *in Inches* ——— ,523598
Content in Inches ——— 21252,7(08778912
Multip. proper ——— ,0025461
Content in A. G. ——— 75,3641 9947
Product found ——— 40589,744
Content of a Sphere whose Axe is 1, *in A. G.* ——— ,0018567
Content in A. G. as before *propè* ——— 75,3629776848

§. *Or by the Table of Areas of Circles.*

Multip. the Area coresp. to the Lesser Diam. by Two thirds of the Axe, the Product is the Content in A. G.

Lesser Diam. 26 per Tab. ——— 1,8827 A. G.
Axe 60,044 whose two thirds is ——— 40,029
Content in A. G. ——— 75,3625983

2 *By the Axe, and Content; to find the Lesser Diam.*

Divide the Content by two Thirds of the Axe the Quote is the Area coresp. to the Lesser Diam, against which in the Table stands the Diam.

40,029) 75,3625983 (1,8827 = 26 Diam.

3. By

3. *By the Lesser Diam. and Content; to find the Axe.*

Divide the Content by the Area corresp. the Lesser Diam. to the Quote is two Thirds of the Axe, the Half of which added to its self gives the Axe.

26=1,8827) 75,362598 3 (40, 029
20, 0145
Sum is the Axe *Propè* ——— 60, 0435

XV. Prop. To find the Content of the Frustum of a Spheroid.

1. *By the Greater Diam. or Diam at Bung, and Depth, or Length of a Spheroidal Tun, or Cask; to find the Spheroids Axe.*

Multip. the Sq. of the Greater Sem-Diam. or Sem-diam at Bung by the Sq. of the Depth or Sem-length, and Divide the Product by the Differ. of Sq. of the Greater and Lesser Sem-diam, the Sq.R. of the Quote is the Sem-Axe.

Sem-dia. Above 50 Sq. 2500 Sem-dia. Above 50 Sq. 2500
Depth ——— 22 Sq. 484 Sem-dia. Below 39 Sq. 1521
Product ——— 1210000 Differ. 979

979) 1210000 (1235,955 Sq. R. 35, 156 Sem-Axe.
2
Spheroids Axe is ——— ——— 70,312

Sem-diam. at B. 13. Sq. 169 Sem-diam. at B. 13 Sq. 169
Sem-length —— 16. Sq. 256 Sem-diam. at H. 11 Sq. 121
43264 Differ. 48

48) 43264 (901, 333 Sq. R. 30, 022 = Sem-Axe
2
Spheroids Axe ——— ——— 60,044

2. *Or you may find the Spheroids Axe more readily thus.*

Divide the Rectangle of the Greater Sem-diam. and Tuns Depth or Sem-diam. at Bung and Casks Sem-length; By the Sq. R. of the Differ. of the sq. of the Greater and Lesser Sem-diam. the Quote is the Sem-Axe.

Sem-diam. { Above 50 Sq. —— 2500
Below 39 Sq. —— 1521
Sq. R. is 31,289 ——— 979
Greater Sem-diam. ——— 50
Depth. ——— 22

31, 289) 1100,000 (35, 156 = Sem-Axe.

Sem-dia. at { B. 13 sq. 169 / H. 11 sq. 121 } Sq. R. 6,9282 — 48

Sem-dia. at B. 13
Sem-Length — 16
Rectangle — 208

6,9282(208,7000(30,022 = Sem-Axe.

2. *By the Axe and Greater Diam. or Diam. at Bung and dry or wet Inches; to find the Diam. of the Liquors Superficies.*

As the sq. of the Spheroids Sem-Axe, is to the sq. of the Greater Sem-Diam. or Sem-Diam at Bung. So is the Rectangle of the Dry and Wet Inches of the Axe, to the sq. of the Sem-Diam. of the Liquors Surface. Whose Sq. R. doubled is the Diameter.

Spheroids Sem-Axe — 35,156 Inc.
Dry Inches given — 8

Spheroids { Dry Inc. 43,156 / Wet Inc 27,156 }

Rectangle of Dry and Wet Inches — 1171. 944336
Sem-Axe. 35,156 Log. doubled — 3,091998
Greater Sem-Diam. 50 Log. doubled — 3,397940
Rectangle of dry and wet Inches 1171,944336 — 3,068905
Sum — 6,466845
Differ. — 3,374847
Sem-dia. at Liq. surface — 48,69 — Half — 1,687423
2
Diam at Liq. surface — 97,38
Spheroids Sem-Axe in Inches — 30,022
Wet Inches above the Bung — 8

Spheroid's { Wet Inches 38,022 / dry Inches 22,022 }

Rectangle of wet and dry Inches — 837,324884
Sem-Axe-30,022 Log. doubled — 2,954822
Sem-diam at Bung 13 Log. doubled — 2,227886
Rectan. of wet and dry Inches 837,32 — 2,922829
Sum — 5,150715
Diff r. 2,195893
Sem-diam. at Liq. surface 12,53 — Half — 1,097947
2
Diam. at Liq. surface. — 25,06

3. By

3. *By the Diam. and Depth of a Spheroidal Tun to find the Content.*

To Double the Area of the Greater Diam. add the Area of the Leſſer Diam. the Sum is the Tripled Area of a Mean Circle, which multip. by one third part of the Tuns Depth, or one third of that Multip. by the Depth, produces the Content in A. G.

Diam. { Above—100 per Tab. doubl. —— 55,7020 A.G.
Below— 78 per Tab ——— 16,9445

Tripled Mean ——— 72,6465
One third part is ——— 24,2155
Depth in Inches ——— 22
Content in A. G. ——— 532,7410

4. *By the Dimentions, Diam, of the Liquors Superficies, and Dry or Wet Inc; to find the Vacuity, or remaining Liquor.*

To Double the Area of the Greater Diam. add the Area of the Diam. of the Liquors Surface, one third part of the Sum is the Area of a Mean Circle, which Multip. by the Dry Inches the Product is the Vacuity, whoſe Compl. to the whole Content is the Remaining Liquor.

Greater Diam. 100 per Tab. ——— { 27,8510
27,8510
Diam. at the Surface of the Liq. 97,38 —— 26,4107
Sum —— 82.1127
One third part is the Area of a mean Circle — 27,3709
Tun's Dry Inches ——— 8
Vacuity in A. G. ——— 218,9672
Remaining Liquor ——— 313,7738
Tun's whole Content in A. G. ——— 532,7410

§. *Or thus without finding the Spheroids Axe or Diam. of the Liquors Surface.*

Divide the differ. of the Areas of the Greater and Leſſer Diam. by the Area Correſp. to the Depth Multip. by 1077,16; then Multip. the Quote by the Sq. of the Dry Inches and Subduct

 the

the Product from the Area of the Greater Diam. the Remainder is the Area of a Mean Circle nearely agreeing with the former; the rest of the operation is the same.

Depth 22 Inc. 1,348 Multip. by 1077,16 produces 1452,01168

Diam. { Above 100 —— 27,8510
Below 78 —— 16,9445

1452,012) 10,9065000 (,00751

Dry Inches 8 Squared ———— 64
Product Subducted ———— ,48064
Greater Diam. 100 Inches Tab ———— 27,8510
Area of a Mean Circle ———— 27,37036

5. *By the Dimentions, Diam. of the Liquors Surface and Vacuity, or remaining Liquor: to find the Dry or, Wet Inshes.*

The difference between the whole Content and remaining Liquor; divided by the Area of a Mean Circle Quotes the Dry Inches which subducted from the Depth leaves the Wet Inches.

Whole Content (*Per Rule* 3) is —— 532,7410 A. G.
Remaining Liquor —— 313,7738

Mean Area (*per Rule* 4) 27,3709) 218,9672 (8 Dry / 14 Wet } Inc.

6. *By the Diam. and Depth: to find the Content at every Foot, half Foot, or Inch by an easie Approximation.*

Calculate two Diam. ~~(by Rules 1, 2, of this Prop.)~~ One at 6 Inches, 3 Inches, ~~or half an Inch~~; and another at 18, 9, or 1, 5 Inches from the Top or Greater Diam. to four or five Decimal places, and insert against them their respective Areas, the Difference between them is a common subducend, which doubled and Subducted from the Content of the second Foot, half Foot, or Inch, the Remainder is the Content upon the Third Foot, half Foot, or Inch; and Trebled and Subducted from the Content of the Third Foot &c. leaves the Content of the Fourth

Fourth Foot, &c. and so on throughout the Depth as in the following Example; then add all together and Multiply the Sum by 12 if you Gauge to every Foot, or by 6 if to every half Foot, and the total of the several Product is the Content, but if you Gauge to every Inch, than the Sum it self is the Content.

Exa. Diam { *Above* 100 / *Below* 78 } *Inches and Depth* 22

Spheroids Sem—Axe 35, 156
Dry Inches ——— 15

Spheroids { Dry Inches 35,656 Log. ——— 1. 1552132
Wet Inches 34,665 Log. ——— 1, 539778
Sem—diam. Above 50 Inches doubled Log. — 3, 397940
Sem—Axe 35, 156 doubled Log. Comp. Arith. — 6, 908002
The Sum *(Rejecting the Radius)* ——— 3, 397852
Sem—diam. at the Liquors Surface 49, 995 — 1, 698926
2
Diam. of the Liquors Surface —— 99,990 —— 27.8454 A.G.

§. *Or you may find the Area Speedily thus.*

Sum of the Log. *(cancelling the Radius)* is —— 3, 397852
Sum of Log. of ,0027851 and 4 is ——— 8, 046800
A. G. 27, 8454 ——— 1, 144752
Spheroids Sem-Axe —35, 156
Dry Inches ——— 1, 5

Spheroids { Dry Inches 36. 656 Log. ——— 1, 564145
Wet Inches 33. 656 Log. ——— 1, 527062
Semi—Diam. at Top 50 Inches Log. doubled — 3, 397940
Sem—Axe 35. 156 doubled Log. Comp. Ar. — 6, 908002
The Sum *(Rejecting the Radius)* ——— 3, 397149
Sem—Diam. at the Liquors Surface 49.9546 — 1, 698574
2
Diam. of the Liquors Surface ——— 99.9092 — 27.8003 A.G.

Depth.	Ar. 1 Inch deep.	Differ.
1	27. 8454	.451
2	27. 8003	.992
3	27. 7101	.1353
4	27. 5748	.1804
5	27. 3944	.2255
6	27. 1689	.2706
7	26. 8983	.3157
8	26. 5826	.3608
9	26. 2218	.4059
10	25. 8159	.4510
11	25. 3649	.4961
12	24. 8688	
	321. 2462	

Depth.	Ar. 1 Inch deep.	Differ.
13	24. 3276	.5412
14	23. 7413	.5863
15	23. 1099	.6314
16	22. 4334	.6765
17	21. 7118	.7216
18	20. 9451	.7667
19	20. 1333	.8118
20	19. 2814	.8529
21	18. 3894	.8920
22	17. 4533	.9362
	211. 5265	
	321. 2462	
	532. 7727	

Note, that from hence by Subduction is made a Practical Table; which shall shew by Inspection how many Gallons &c. remain in this Tun at any Number of Dry Inches.

XVI. Prop. to find the Content of a Copper and its Crown.

1. *For the upper part.*

With a sliding Rule take the mean Diameters in the midst of every half Foot of the Depth (*Making a proportional Allowance if you measure on the side Line*) as directed Prop XII. Rule 2.

In the following *Exam.* the first Mean Diam. is 103, the second 100, the third 94, the fourth 90, the fifth 86, 5 now the last Segment will want of half a Foot, therefore draw a Line to the Top of the Crown, and measuring from the last Diam. to this touch Line you will find it 6 Inches, so that the Altitude of this Segment is 3 Inches; Lastly in the Midle of it *viz.* 4, 5 Inches from the Last Diam. or 1, 5 Inch from the top of the Crown the mean Diam. is 81; And thus you have all the Meanes above the Top of the Crown; Then insert against each respective Diam. taken out of the Table of Areas of Circles its respective Area, and Multip. them severally by 6.

Altit

	Altit.	M	D	Ar. 1 Inch deep.		Ar. 6 Inches deep.	
	6	103		29.	5471	177.	2826
	6	100		27.	8510	167.	1060
	6	94		24.	6091	147.	6546
Copper. 42 33	6	90		22.	5593	135.	3558
	6	86.	5	20.	8388	125.	0328
	3	81		18.	2730	54.	8190
	9	75				78.	1182
	42					885.	3690

2. *For the Crown.*

Let fall two Perpend, from the Top to the Bottom each 42 Inches, the Diſtance between them is equal to the Diam. at Baſe 75 Inches, for you cannot take it Actually; Then let fall a Perpend. to the Top of the Crown which will be 33 Inches, which Subducted from 42 the whole Altitude the Remainder is 9 Inches the Crown's Altitude.

If the *Crown* have equal Curvature at the Top and toward the Bottom it may be Gauged as the *Fruſtum of a Sphere.*

But if more Curvature at Top then towards the Bottom then as the *Fruſtum of a Parabolical Conoidas in Prop.* VII. *Rule* 23.

75 Inches Diam at Baſe per Tab. ——— 15,6662 A.G.
Half the Crown's Altit. 4,5 Inches ——— 4,5
Crown's Cont. in A.G. ——— 70,49790

3. *For the Lower part.*

Take an Arith. mean between the Diam. at Top of the Crown, and Diam at Baſe, and Multip. the Area Correſp. thereto by the Altitude of that part, the Product is the Content in A.G. from which Subduct the Crown's Content the Remainder inſert under the other, then the Sum is the whole Content.

Diam. at { Top of the Crown—79 / Baſe——— 75 } Inches

Sum—154
Half—77 per Tab. 16,5129 A. G.
Segment's Altitude ——— 9
Cont. of the Segm. or Lower part ——— 148,6161
Crown's Content. Subd. ——— 70,4979
Remainder ſought is ——— 78,1182

XVII. To

XVII. Prop. To find the Content of any Square or Round Mash-Tun.

Enter the proper Table with the Mean Sq. or Diam. of the goods usually Wet, and against it stands the Content in Gallons &c. upon 1 Inch, which Multip. by any assigned Altitude produces the Content.

Note that two Gallons Dry measure make a Peck, four Pecks a Bushel, and Eight Bushels a Quarter.

In 1678 when I was Supervisor in Newcastle, Bishoprick *of* Durham *&c. Places well known to many practical Gaugers I made some experiments of several sort, of Malt, and found that the usual sort, and of ordinary Grinding when the Worts had passed thro it would not be contracted above one Eight part, I thought it therefore convenient to make the Divisor for Square Mash-Tuns* 238, *and for round* 303, *which may indifferently serve for most of the Northern Counties; but when all is said this must be left to the Practical Gauger, who must frame his Divisors according to the Country Malt made use of.*

The Square therefore of any Side Multiplied by, 0042 the Quote of 1, divided by 238 is the Construction of the Table for Square Tuns; Or the Square of any Diameter Multiplyed by ,0033, the Quote of 1, Divided by 303. is the Construction of the Table for Round Tuns.

And either of these Tables having their Second Differences equal may be made by an easie Collection, an Example is needless in a Case so Obvious.

XVIII. Prop. To Gauge the Worts, in any Back or Cooler.

Divide the Sum of the Wet Inches taken in all the places *(which in large Backs ought to be every Five or Six Feet Square)* by the Number of Places they were taken at, the Quote is the Depth.

A Gauger finds in several Places the Wet Inches— { 4,8 / 5,2 / 4,4

4

Number of Places—— 4) 18,4 (4,6 Inches Depth.

CHAP. III.

CHAP. III.

Of Cask-Gauging by the Tables, either the whole or the Parts, with some Propositions for the better accomplishing the Practical Gauger.

Prop. I. To find the whole Content of a Cask in A.G. under any given Notion, &c.

1. *Of a Cylindrical Cask.*

1. *By the Diam. and Length; to find the Content in A. G.*

Multip the Area corresp. to the Diam. by the Length, the the Product is the Content in A. G.

Diam. at Base 26 per Tab. of Areas of Circles 1, 8827 A.G.
Length in Inches ——— 32
Content in A. G. ——— 60, 2464

2. *By the Diam. and Content; to find the Length.*

Divide the Content by the Area corresp. to the Diam. and the Quote is the Length.

1, 8827) 60, 2464 (32 — Length

3. *By the Content, and Length; to find the Diameter.*

Divide the Content by the Length, and with the Quote enter the Table against which in the proper Column Stands the Diam.

Length —32) 60, 2464 (1, 8827 — 26 Diam.

4. *By the Diam. and Length; to find the Diagonal.*

To the Sq. of the Diam. add the Sq. of the Sem-length, the Sq. R. of the Sum is the Diagonal.

Diameter 26 Sq. ——— 676
Sem-Length 16 Sq. ——— 256
Sq. R. —— 30, 53 is the Diagonal ——— 932

5. *By the Diagonal, and Diam.; to find the Length.*

From the Sq. of the Diag. subduct the Sq. of the Diam. the Sq. R. of the Remainder is the Sem-length.

Diagonal 30. 53 Squared ——— 932
Diameter 26 Squared ——— 676
Sq. R. — 16 is the Sem Length ——— 256

6. By the Diagonal, and Length to find the Diameter.

From the Sq. of the Diag. Subduct the Sq. of the Semi-Length, the Sq. R. of the Remainder is the Diam.

Diagonal 30, 53 Squared ——— 932
Sem-length 16 Squared ——— 256
Sq. R. 26 is the Diameter ——— 676

2. *Of a Spheroidal Cask. &c.*

1. By the Diam. and Length; to find the Content in A. G. if the Cask be esteemed the midle Frustum of a Spheroid. &c.

To double the Area of the Bung Diam. add the Area of the Head Diam. and Multip. the Sum by One Third part of the Length.

Or One Third of the Sum Multip. by the whole Length the Product is the Content in A. G.

Area of the Diameter at — Bung 26 Inches per Tab. — 1,8827 A.G. / 1,8827
Head 22 Inches ——— 1,3480

Sum is the Tripled Area of a mean Circle ——— 5,1134
The One Third part thereof is ——— 1,7045
Length in Inches ——— 32
Content in A. G. *as Spheroidal* ——— 54,5440

2. By the Diam. and Length to find the Content in A. G. if the Cask be esteemed the Midle Frustum of a Parabolick Spindle. &c.

To double the Area corresp. to the Bung Diam. add the Area of the Head Diam. and from the Sum Subduct four times the Area against the Differ. of Diam. the Remainder is the Tripled Area of a Mean Circle, the rest of the Operation is the same as in *Rule* 1.

Area of the Diameter at — Bung 26 Inches doub. ——— 3,7654 A.G.
Head 22 Inches ——— 1,3480
Sum — 5,1134

4 Differ. of Diam. —,0446 Multip. by 4. ——— 1784
Rests Tripled Area of a mean Circle ——— 4,9350
One third part of which is ——— 1,645
Length in Inches ——— 32
Content in A. G. as *Parabolical* ——— 52,640

3. By

3. *By the Diam. and Length to find the Content in A. G. if the Cask be esteemed the Midle Frustum of two Cones, abutting upon one Base &c.*

To the Area of the Bung Diam. add the Area of the Head Diam. and from the Sum Subduct One Third part of the Area Area Corresp. to the Diff.r.of Diam. the Remainder is the Doubled Area of a Mean Circle, which Multip. by the Sem-length produces the Content in A.G.vid. Prop.XII. Chap.II.

Area of the—Bung 26 per Table ——— 1,8827 A.G.
Diameter at—Head 22. ——— 1,3480
Sum—3.2307
4. Dif. of Diam.—,0446 whose one third is ——— ,0139
Doubled Area of a Mean Circle ——— 3,2168
Sem-length in Inches ——— 16
Content in A. G. as *Conical* ——— 51,4688

4. *By the Diam. and Content; to find the Length.*

Divide the Content by the Mean Area *(found by Rule 1.)* the Quote is the Length.

Mean Area as Spheroidal - 1,7045) 54.5440 (32—length.

5. *By the Content, Length, and One Diam; to find the other.*

Divide the Content by the Length, the Quote is a Mean Area, which Triple, and Subduct from it the Area of the Diam. at Head, the Remainder is the doubled Area of the Diam. at Bung: Or from the Tripled Area Subduct the doubled Area of the Bung Diam. there rests the Area of the Head Diam. with either of which enter the Table, and against it stands the Diam. sought.

Length-32) 54 5440(1,7045 A.G. mean Area.
5,1134 Tripled Area
1,3480 Area of Diam. at Head 22 Inches
3,7654 Doubled Area of Bung Diam.
Half—1,8827——26 Inches Diam. at Bung

6. *By the Length, and Diam; to find the Diagonal.*

To the Sq. of the Sem-Sum of Diam. add the Sq. of the Sem-length, the Sq. R of the Sum is the Diagonal.

Sem-ſum of Diam. 24 Squared ——— 576
Sem-length —— 16 Squared ——— 256
Sq.R. is the Diag. 28-844 ——— 832

7. *By the Diagonal, and Diam; to find the Length.*

From the Sq. of the Diag. Subduct the Sq of the Sem-ſum of Diam. the Sq.R. of the Remainder is the Sem-length.

Diagonal —— 28,844 Squared ——— 832
Sem-ſum of Diam. 24 —Squared ——— 576
Sq. R. is the Sem-ſum 16 ——— 256

8. *By the Diam. and Length; to find the Hypoth. or Slope Side.*

To the Sq. of the Sem-length, add the Sq. of the Sem-differ. of Diam. the Sq. R. of the Sum is the Hypothenuſe.

Sem-length — 16 Squared ——— 256
Sem-dif. of Diam. 2 Squared ——— 4
Sq.R. Hypoth. —16. 125 ——— 260

9. *By the Hypothenuſe and Diam; to find the Length.*

From the ſq. of the Hypoth. Subduct the ſq. of the ſem-differ. of Diam. the ſq.R. of the Remainder is the Sem-length.

Hypothenuſe—16.125 Squared ——— 260
Sem-dif. of Diam. 2—Squared ——— 4
Sq. R. Sem-length 16 ——— 256

10. *By the Diagonal, Length, and Hypoth; to find the Diameters.*

From the Sq. of the Diag. Subduct the Sq. of the Sem-length, the Sq.R. of the Remainder is the Sem-ſum of the Diam.

From the Sq. of the Hypoth. Subduct the Sq. of the Sem-length the Sq R. of the Remainder is the Sem-differ. of Diameters.

Sem-ſum { more / leſſe } the Sem-differ, is the { Greater / Leſſer } Diameter.

Diagonal 28, 844 ſq. 832 — Hypoth. 16.125 ſq. 260
Sem-len. 16 —— ſq. 256 — Sem-len. 16 —— ſq. 256
Sq.R. —24 Sem-ſum. 576 — Sq.R —2 ſem-dif. 4

Sem-ſum 24 { more 2—26 Bung / leſſe 2—22 Head } Diameter.

11. By

11. *By the Diagonal, Length, and one Diam; to find the other.*

From the *Sq* of the Diag. *Subduct* the ſq. of the ſem-length, the ſq. R. of the Remainder is the ſem-ſum of the Diam. which double and ſubduct from it the Diam. there reſts the Diam. ſought.

Diag. 28.844 ſquared ——— 832
Sem-len. 16—ſquared ——— 256

Sq. R.—24 is the ſem-ſum of Diameters —576

2

48 Diam. at Head and Bung
26 Diam. at Bung ſubd.

Reſts 22 Diam at Head

Prop. II. Of a Cask part full whoſe Axe is Parallel to the Horizon.

1. *By the Whole Content, Bung. Diam. and Dry or Wet Inches; to find the Ullage, or Remaining Liquor by the Table of Segments.*

To the *Dry or Wet Inches* given, add Three Cyphers, and Divide it by the *Bung Diam.* the quote is a *Verſed Sine*, which found in the Table under V. *S.* againſt it in the next Column ſtands a *Segment*, which Multip. by the *whole Content* the Product is the *Ullage, if the Dividend were the Dry Inches, or Remaining Liquor if it were the Wet Inches.*

2. *By the whole Content, Bung Diam. and Ullage, or Remaining Liquor; to find the Dry or Wet Inches.*

To the *Ullage, or Remaining Liquor* add Four Cyphers, and Divide it by the *whole Content*, the Quote is a *Segment*, which found in the Table under *Seg.* againſt it ſtands a *Verſed Sine*, Which Multip. by the *Bung Diam.* the Product is the *Dry Inches if the Dividend were the Ullage, Or Wet Inches if it were the Remaining Liquor.*

Exam.

Exam. 1. A Cask's Bung Diam. 26 Inches and whole Content as Spheroidal 54.544 A.G. what is the Ullage when 7.8. Inches are Dry.

B.D. 26) 7,800 (,300 = ,2523 Segment

Whole Content as *Spheroidal* —— 4,544 A.G.

Ullage in A. G. —————— 13,7614512

The Converse.

Cont. 54,544) 13,7614512 (,2523 per Tab—,300 *V.S.*

Bung Diam. In Inches ———————— 26

Dry Inches, are ———————— 7,800

Exam. 2. A Cask's Bung Diam. 25 Inches and whole Content as Parabolical 52,64 A.G. What is the remaining Liquor when the Wet Inches are 18.2.

B. D. 26) 18,200 (,700 *V.S.* per Tab-,7477 *Seg.*

Whole Content as *Parabolical* ———— 52,64

Remaining Liquor ———————— 39,3589 28

The Converse.

Cont. 52,64) 39,3589 28 (,7477 *Seg.* per Tab. ——,700 *V.S.*

Bung Diam. in Inches ———————— 26

Wet Inches are ———————— 18,200

And thus you may find the Ullage, or Remaining Liquor, and the Converse, under any given Consideration, without finding the Content, or Correspondent Frustum as Spheroidal.

Prop. III. Of a Cask part full whose Axe is Perpendicular to the Horizon.

1. *By the Whole Content, Bung Diam. Diam. of the Liquors Surface, and Dry or Wet Inches; to find the Ullage or Remaining Liquor.*

To Double the Area of the Bung Diam. add the Area of the Diam of the Liquors Surface, One third part thereof is the Area of a mean Circle; Or to two Thirds of the Area of the Circle at Bung, add one Third of the Area of the Circle at Head

Head, the Sum is the Area of a Mean Circle, Which Multip. by the Wet Inces Above, or Dry Inches Below the Bung, and the Product added to, Or Subducted from the Sem-Content gives the Content of the Greater of Leſſer Fruſtum.

2. *By the Whole Content, Bung Diam. Diam. of the Liquors Surface, and Ullage, or Remaining Liquor; to find the Dry, or Wet Inches.*

The Differ. between the Sem-content, and Remaining Liquor is the Replete Fruſtum Above, or Vacant Fruſtum Below the Bung, which divided by the Area of a Mean Circle Quote the Wet Inches Above, or Dry Inches Below the Bung, Which added to, or Subducted from the ſem-length, gives the Wet or Dry Inches ſought.

Diam. at { Bung —— 26 doubled ———— 3,7654 A. G.
{ Liq. Surface 25.06 ———— 1,7490

Sum — 5,5144

One Third is the Area of a Mean Circle — 1,83813
Wet Inches above the Bung ———— 8
Content of the Fruſtum above the Bung — 14,70504
Cask's ſem-content ———— 27.272
Remaining Liquor ———— 41,97704
Ullage therefore is ———— 12,56696
Whole Content in A. G. ———— 54,544

The Converſe.

Remain Liquor. 41,97704 A. G.
Sem-Content — 27,272

1,83813) 14,70 04 (8 = Wet Inc. above the Bung.

§. *Or by the Dimentions without the Axe, or Diam. of the Liquors Superficies,*

Divide the differ. of the Areas of the Head, and Bung by the Area Correſp. to the ſem-length Multip. by 1077.161. and Multip the Quote by the Sq. of the Wet Inches above, or Dry Inches below the Bung, and ſubduct from the Area of the Bung Diam. the Remainder is the Area of a mean Circle; Which Multip. by the Wet Inches above the Bung, and the Product added to the ſem-content, the ſum is the remaining Liquor.

Or

Or the Mean Area Multip. by the Dry Inches below the Bung, and the Product ſubducted from the ſem-content leaves the Remaining Liquor, whoſe Comp. to the whole Content is the Ullage.

16 Sem-len = ,713 Multip. by 1077, 161 gives 768,015793

Diam at { Bung 26 — 1,8827 / Head 22 — 1,2480

768,016) ,53470,000 (,000696

Wet Inches above the Bung 8 Inches ſq — 64

Product ſubd.	,044544
Bung Diam. 26 per Table	1,8827
Area of a Mean Circle	1,838156
Multip. by the wet Inches above the Bung	8
Product add	14,705248
Sem-Content	27,272
Remaining Liquor	41,977248
Ullage Or Vacuity.	12.566752

Cask's whole Content in A. G. — 54,544

§ *Note*, that the Duty of Syder, and other Exciſeable Liquors being to be Returned according to the *Wine Gallon*, and there being not any Tables for W. G. in this Book it will be neceſſary to ſhew the Country Gauger how to Reduce *Wine Gallons, to Ale Gallons, and the contrary.*

1. *To Turn A. G. into W. G.*

Multip. the A. G. by 282, and divide the Product by 231 (*the Cube Inches in a Wine Gallon now in uſe*) the Quote is the W. G.

Or Multip. the A. G. by 1,22078 the Product is the W. G. Seperating as many Figures towards the right hand as there are places of Decimals in both Multipliers.

2. *To Turn W. G. Into A. G.*

Multip. the W. G. by 231, and divide the Product by 282 (*the Cube Inches in an A G. now in uſe*) the Quote is the A. G.

Or Multip. the W. G. by,81923 the Product is the A. G. Seperating as many figures as are *required by Decimal Arithmetick.*

A. G.

A. G. in a Cask ——— ——— 54,544
Quote of 282 Divided by 231 is ——— 1,22078

Product is the W. G. ——— 66,586224,2
W. G. in a Cask ——— 66,586
Quote of 231 Divided by 282 ——— ,81915

Product is the A. G. ——— 54,543921 90

CHAP. IV.

Of the Definition and Nature of Logarithms, with the Description, Use, and Construction of Mr. *Brigg's* Tables, Printed in my *Guide for the Practical Gauger*, here inserted at the request and for the satisfaction of some of my peculiar Friends who are very desirous to understand the Excellency of this Artificial kind of Arithmetick.

§. 1. *Of the Definition, and Nature of Logarithms.*

Logarithms are Numbers that differ *Arithmetically* as the *proportional Numbers* to which they are fitted differ *Geometrically*; And perform the same by *Addition* and *Subduction*, which natural Numbers effect by *Multiplication* and *Division*, whereby the most troublesome part of Calculation is avoided.

Now every *Logarithm* is divided into *Two parts*.

The *First* is called the *Characteristick* or *Index*, which shews of how many places the Number sought doth consist, or how far from Unity the first Figure of any Number stands, and is alwayes to be seperated from the rest by a Point, or Comma, the rest of Figures more exactly shew the Number signified thereby.

Thus the *Charact. or Index* of Unity is 0, of Tens is 1, of Hundreds is 2, of Thousand is 3, and so to proceed further the Charact. is alwayes an Unit less than the Places or figures of the given Number.

	5	4	3	2	1	0
As in this *Exam.*						
	6	8	7	3	2	5

the *Charact* or *Index* of 5 is 0, of 3 is 2, of 8 is 4, of 6 is 5.

§ 2.

§. II. *The Description of the Tables of Logarithms.*

The First Page of the Tables consists of six Columns, those three (*under N*) contain all the Numbers from 1, to 99 successively, and in the other three (*under Log.*) against each Number is placed its respective Logarithm with its proper Charact, or Index preponed. After which there followeth another Table. (*entituled the Table of Logarithms*) which contains the Logarithms of all the whole Numbers under 1000 consisting of twelve Columns *viz.* Six in the left and six in the right hand Pages.

In the First of these Columns (*marked at the top of each Page N*) is inserted all the whole Numbers from 100 to 1000 in their Natural order.

In the next Ten Columns (*marked at the Top* 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.) are set down Logarithms without their Characteristicks. And the Twelfth Column (*under L*) contains the common differences of those Logarithms.

§ III. The Use of the Tables of Logarithms in finding the Logarithm of any Number, and the Number of any Logarithm.

1. *To find the Logar. of any Number consisting of One or Two Places.*

Seek the given Number in the first Page of the Table (*under N*) and directly against it in the next column (*under Log.*) is set down the Logarithm with its proper Charact. or Index.

Logar. corresp to *Number* $\left\{\begin{matrix}3\\36\end{matrix}\right\}$ is $\left\{\begin{matrix}0.477121\\1.556303\end{matrix}\right\}$

2. *To find the Logar. of any Number consisting of three Places.*

Seeke the given Numbers in the first column of the Tables (*entituled the Table of Logarithms*) under N, and against it in the next column under 0, stands a Number to which prepone the proper Charact. (*found by* §. 1.) and you have the Logarithm required.

Logar. corresp to *Number* $\left\{\begin{matrix}125\\150\end{matrix}\right\}$ is $\left\{\begin{matrix}2.096910\\2.176091\end{matrix}\right\}$

3. To

3. *To find the Logar. of any Number consisting of four Places.*

Seek the three First Figures of the given Number in the first column (*under N*) as before, and the fourth Figure at top, under which in that column directly against the three first Figures stands a number, to which prepone the proper Charact. and you have the Logarithm required.

Logar. corresp. to *Number* $\left\{\begin{matrix}1250\\1257\end{matrix}\right\}$ is $\left\{\begin{matrix}3.096910\\3.099335\end{matrix}\right.$

4. *To find the Logar. of any Number consisting of Five, or Six Places.*

Seek the Logarithm of the four First Figures of the given Number (*by Rule 3*) then take the common Difference (*under D*) correspondent to the three First Figures, and multiply it by the last Figure or Figures, the Product seperating as many Figures towards the right hand as you multiplied by, add to the Logar. before found and the proper Charact. preponed gives the Log. required.

Let the Number given be 12572, and the Log. thereof required.

Seek 125 the three First Figures (*under N*) and 7 the Fourth figure at Top, and in the common Angle stands 099335, and the common Difference (*under D*) is 346, which Multip. by 2 the last figure the product is 692, seperate the last figure and add the residue 69 to 099335, the sum is 099404 to which prepone 4 the proper Charact. and the Logar. will be 4.099404 corresp. to 12572.

Again let 125724 be the Number given, and the Log. required.

The Logar. of 1257 (*besides the Charact.*) is 099335, and the common Differ. 346 which Multip. by 24 the Product is 8304, cut off the two last figures towards the right hand and add the Remainder 83 to 099335, the sum is 099418 to which prepone 5 the proper Charact. the Log. is 5,099418 corresp. to the given number 125724.

5. *To find the Logarithm of any mixt Number.*

Suppose the Number given a Whole Number, and find the Logar. thereof without the Charact. by the preceding Rules to which prepone the proper Charact. or Index. (*viz. an unit lesse then the places of the whole part of the given Number*) the Sum is the Logar. required.

Note that in whole Numbers the Charact. or Index proceeds from the Right hand to the Left, but in Decimals it proceeds from the left to the Right.

Thus the Charact. of Unity is 0, of Tens is $\bar{1}$, of Hundreds is $\bar{2}$, of Thousands is $\bar{3}$, as in this Number 3,5781, the Charact. of 3 is 0, of 5 is $\bar{1}$, of 8 is $\bar{2}$, of 1, is $\bar{3}$; Or rather as the distance of their places is from Unity take their Complement to 10, which will make Addition and Subduction more plain and easie, as instead of $\bar{1}$ take 9, of $\bar{2}$ take 8, of $\bar{3}$ take 7 and so forwards makeing small strokes over their heads to signifie they are defective.

In the Six following Examples the pure parts of the Logarithm are the same viz. 662228 but the *Charact.* or *Index* alters according to the distance of the first significant figure from Unity as before premised.

Note that it is the nature of these Logar. let the number given be great or small, whole or mixt, to consist of the same number of places.

578,4 —	2,762228	,5784 —	$\bar{1}$,762228
57,84 —	1,762228	,05784 —	$\bar{2}$,762228
5,784 —	0,762228	,005784 —	$\bar{3}$,762228

The Converse.

6. *To find a Number consisting of one, or two places correspondent to any given Logarithm.*

Seek in the First Page of the Tables the given Logarithm (*under Log.*) add directly against it in the next column (*under N*) is set down the number sought.

Number corresp. to *Logar.* $\left\{\begin{matrix}0\,47712\\ \bar{1}.556303\end{matrix}\right\}$ is $\left\{\begin{matrix}3\\ 36\end{matrix}\right.$

7. *To find a Number consisting of three places correspondent any given Logarithm.*

Seek the rest of the Logar. (*without the Characteristick*) in the second column of the Table of Logarithms (under 0) and the Number against it in the first column (*under N.*) is the Number sought.

Number corresp. to *Logar.* $\left\{\begin{matrix}096910\\ 176091\end{matrix}\right\}$ is $\left\{\begin{matrix}125\\ 150\end{matrix}\right.$

8. To

8. *To find a Number consisting of four places correspondent to any given Logarithm.*

Seek the rest of the Logar. (*besides the Charact.*) in any of the columns (*marked at Top* 0,1,2,3,4,5,6,7,8,9,) then to the three First figures in the first column postpone the figure at Top directly over the given Logar. and the Sum is the Number sought.

Number corresp. to *Logar.* $\left\{\begin{matrix}096910\\099335\end{matrix}\right\}$ is $\left\{\begin{matrix}1250\\1257\end{matrix}\right.$

9. *To find a Number consisting of Five, or Six places correspondent to any given Logarithm.*

Find the next lesser Logar. (*without the Charact.*) in any of the columnes (*marked at Top* 0,1,2,3,4,5,6,7,8,9) and to the three First figures in the first column postpone the Figure at Top which reserve; then from the given Logar. Subduct the next lesser Logar. to the Difference postpone 9, if the Number must consist of five places, or 99 if it must consist of six places, and divide it by the common difference (*under D*) directly against the next lesser Logar. Lastly postpone the Quote to the four reserved Figures, and the Sum is the Number sought.

Let 099404 *be the given Lgarithm, and the Numer sought to consist of Five places.*

The next Lesser Logar. in the Tables is 099335, over which at Top stands 7 wherefore 125 in the first Column, and 7 at Top are the four First Figures of the Number sought, then deduct 099335 out of 099404 the given Logar. and the difference is 69 to which postpone the Figure 9 it makes 699, which divide by 346 the common differ. the Quote is 2, therefore 12572 it the Exact Number correspondent. to 099404 the given Logarithm.

Again let 099418 *be the given Logarithm, and the Number sought to consist of Six places.*

The next Lesser Logar. is 099335, against it the first column stands 125 and directly over it at Top 7; so that the four first Figures are 1257, then the differ. of the given Logar. 099418 and the next lesser 099335 is 83 to which postpone 99

it makes 8399 which divided by 346 the common differ. quotes 24, so that 125724 is the Number corresp. to the given Log. 099418.

Then if it be a *Perfect Logarithm* (*that is*) the Logar. of a Number above 1, observe of how many Units the Charact. or Index of the given Logar. doth consist, and seperate toward the left hand one more of the figures so found (*as if the Charact. be* 0 *Seperate* 1, *if it be* 1 *seperate* 2, *if* 2 *seperate* 3 &c.) according to §. 1. which Figures so seperated are the *whole Numbers*, and if any Figures remain towards the right hand they are a *Decimal Fraction*.

Numbers corresp. to Logar. { 0,099404 / 1,099404 / 2,099404 } is { 1,2572 / 12,572 / 125.72

But if it be a *Defective Logarithm* (*that is*) the Logar. of a Number lesse then 1, observe the Note at the end of *Rule* 5.

§. IV. The Use of the Tables of Logarithms in Arithmetick.

1. *Addition of Logarithms.*

When the *Charact.* or *Indices* are both, or all of *Integers* or whole Numbers, add them together as in common Arithmetick; When the Charact. are some of Integers and *Some of parts*, and the Sum of the Charact. upon the Addion be 10 or above, cast away 10, and the Remainder is the Charact. of *Integers*, if under 10 of *Decimal parts*;

When the Charact. or Indices are both or all of *Decimal parts*, and if added be under 10, add 10 to them, if exactly 10 then add Unity, if above 10 then cast away 10, and the Charact. is of *Decimal parts*.

1,177121	2,05̄5935	8,237242	9̄,397940
1,397940	7,583199	9,875062	9,875061
2,875061	9̄,540104	0811275	9,273001

2. *Subduction of Logarithms.*

When the Charact. or Indices are of *Integers* Subduct as in common Arithmetick;

When the Charact. are either of them, or both of *Decimal Parts* ſet them one above another, then if the Charact. of the Upper be a ſmaller figure then the Lower add 10 to it, and if the Upper be of greater value then the Lower (*that is a Larger Charact. by place*) the Remainder is *Integers*; if not *Decimal parts.*

3,572141	2,033421	$\bar{2}$,875062	$\bar{2}$,875062
1,235781	$\bar{2}$,875062	2,033421	$\bar{2}$,574032
2,336360	2,158359	7,841641	$\bar{1}$,301031

3. *Multiplication by Logarithm.*

To *Multiply* one Number by another, is to add their Logar. and the Sum is the Logar. of the *Product.*

N	Log.	Log.
Multiplicand 30	1,477121	5,12 — 0,709270
Multiplier — 25	1,397940	1,55 — 0,190332
Product — 750	2,875061	7,960 — 0,899602
30 — 1,477121	,75 — $\bar{1}$,875061	144 — 2,158363
2,5 — 0,397940	,125 — $\bar{1}$,096910	,75 — $\bar{1}$,875061
75 — 1,875061	,09375 — $\bar{2}$,971971	108,— 2,033424

Note that in every *Multiplication* there are four mean Proportional Numbers viz. As 1 is to the *Multiplyer*, ſo is the *Multiplicand*, to the *Product*; Now becauſe 1 (*which is the firſt proportional*) doth neither multiply, nor divide; and the Logar. of 1 being 0,000000, it is manifeſt therefore that the Sum of the Logar. of the *Multiplicand* and *Multiplyer* is equal to the Logar. of the Product.

4. *Diviſion by Logarithms.*

To Divide one number, by another is to Subduct the Logar. of the *Diviſor* from the Logar. of the *Dividend*, the Remainder is the Logar of the *Quote.*

N	Logar.	N	Logar.
Dividend. — 750	2,875061	7,936	— 0,899062
Diviſor — 25	1,397940	1,55	— 0,190332
Quote — 30	1,477121	5,12	— 0,709270
75 — 1,875061	,09375 — $\bar{2}$,971971	108	— 2,033424
2,5 — 0,397940	,125 — $\bar{1}$,096910	,75	— $\bar{1}$,875061
30 — 1,477121	,75 — $\bar{1}$,875061	144	— 2,158363

N

Note that in every *Division*, As the *Divisor* is to 1, so is the *Dividend* to the *Quote*; now because 1, is alwayes one of the four mean Proportionals, the Logar. of the *Dividend* notwithstanding the Addition of 0,000000 (*the Logar. of* 1) is the same without any alteration, Hence it is evident that the Sum of the Logar. of the *Divisor*, and *Quote* is equal to the Logar. of the *Dividend*.

5. *The Single Rule of Toree Direct.*

In the Rule of three Direct, or Reverse, the First and Third Termes, and the second and Fourth must allwayes be of the same Denomination, whether Moneys, Weights, Measures, &c. and if compounded they must be first be reduced to the lowest of the given Denominations; And the given Termes must he so collocated that that which constitutes the Question (*which commonly followes these, or such like words, How many? How much? What will? How long? How Far?*) must be set in the Third place, and that of the same denomination in the First, & the remaining Number in the Second; Then in the Rule *Direct*, as the First Term is to the Second, so is the Third to the Fourth: Add therefore the Logar, of the Second and Third Terms, and from the Sum Subduct the Logar. of the First the remainder is the Logar. of the Fourth.

Exa. As 6. 12 : so 216 432.

As 6 ——0,778551	6 Comp. Ar.——9,221849
to 12——1,079181	12——1,079181
So 216 —2,334454	216 ——2,334454
Sum—— 3,413635	432——2635484
to 432— 2,635484	

Or you may take the Arith. Compl, of the Logar. of the First Term, and add the three Logar. the Sum abating the Radius is the Logar of the Fourth Term as above.

Note that if the Sum of the Logar. of the First and Fourth be equal, to the Logar. of the Second and Third, the work is tru otherwise erroneous.

6. *The Single Rule of three Reverse.*

Place the Numbers as in the Rule *Direct*; then as the First Term is to the Third, so is the Second to the Fourth: Add therefore

therefore the Logar. of the First and Second Terms, and from the Sum Subduct the Logar. of the Third Term, The Remainder is the Logar. is the Fourth.

Or you may take the Arithm. Compl. of the Logar. of the Third Term and add the three Logar. the Sum rejecting the Radius is the Logar. of the Fourth Term.

Exa. As 432 to 6 So is 216 to 12.

432	2,635484	432	2,635484
6	0,778151	6	0,778151
Sum	3,413635	216 Ar. Comp.	7,665546
216	2,334454	12	1,079181
12	1,079181		

Note that if the Sum of the Logar. of the Third and Fourth Terms be equal to the Logar. of the First and Second the worke is true.

A General Rule to know whether the Question belong to Rule Direct. or Reverse.

If the Third { *Greater* } *then the First and* { *Greater* } *then the*
Term be { *Lesser* } *answer required* { *Lesser* } *Second*
then it belongs to the Rule *Direct.*

But if the Third { *Lesser* } *then the First* { *Greater* } *then the*
Term be { *Greater* } *and answer required* { *Lesser* } *Second*
then it belongs to the Rule *Reverse.*

I might here shew the excellent use of the Logar. in the resolution of Questions belonging to the *Double Rule of Three, Fellowship, Alligation,* and *Compound Interest,* having lying by me some Propositions, and Tables relating thereto, which I question not but would be very acceptable to the Legitimate Sons of Art, whose favourable construction of this Edition may be a means to encourage me to another Attempt of this nature.

7. *Any Number being given, to extract the Root thereof.*

In any Rank of Numbers continually proportional from 1, the Logarithm of any of them being divided by the Charact. or Index of the power sought quotes the Logar. of the Root.

Thus

Thus to extract the Square Root is to Halve it or divide it by 2, to extract the Cube Root is to divide it by 3, the Bidrate by 4, and so forwards.

Charact.	Powers	Geom. Mea	Logarithms
		1	0. 000000
1	Root	2	0. 301030
2	Square	4	0. 602060
3	Cube	8	0. 903090
4	Biquadrate	15	1. 204120
5	Surfolid	32	1. 505150
6	Squared Cube	64	1. 806180
7	Bisur Solid	128	2. 107210
8	Square Sq. Square	256	2. 408240
9	Cubed Cube	512	2. 709270
10	Sq Sur. solid.	1024	3. 010300
11	Cube Sur. solid	2048	3. 311330
12	Sq. Sq. Cube	4096	3. 612360

The Converse.

The Logarithm of the Root being Multiplied by the Charact. or Index of any given Power, the Product is the Logar. of that Power, as to Square any Number is to multiply its Logar. by 2, to Cube it by 3, to Biquadrate it by 4, and so forwards.

Number 4 Logar. 0,602060 Number 8 Logar. 0,903090
Divided by 2) 0,301030 Divided by 3) 0,301030
Square Root is 2. Cube Root is 2.

Note if you cannot find the Numbers exactly in the Tables then use Decimal Fractions which in these kind of operations are most easy.

§. V. The Construction of the Tables of Logarithms.

To make a Table of Logarithms you must propose unto your self a convenient Rank of Numbers Geometrically proportional, and assign unto them for Logarithms any Numbers at pleasure, provided that the Logarithms so assigned differ

among

among themselves by Arithmetical proportion as their correspondent Numbers differ by Geometrical proportion: Now the Numbers continually proportional which Mr. *Briggs* hath made choice of as most fit for Use in Arithmetical operations are these ———

viz.	1	to which he hath assigned for Logarithms ——	0,000000
	10		1,000000
	100		2,000000
	1000		3,000000
	10000		4,000000
	100000		5,000000

To make the Logarithms of the intermediate Numbers.

1. Find so many continual means between 10 and 1, or any other of the assigned numbers (*that is*) between 100 and 10, between 1000 and 100, till that which come neerest to 1,10, or 100 may be a mixt Number less then 2,20, or 200, and so neer 1,10, or 100, that it may have seaven Cyphers before the significant Figures.

To 10 therefore it being the least so most apt for this purpose, postpone Fourteen pairs of Cyphers and extract the *Square* Root, which you will find to be 3,162277660168 38, to which postpone seven pairs of Cyphers more and extract the *Sq. R.* thereof which is 1,778279410038 92, and so successively by postponing seven pairs of Cyphers you are to produce Twenty seven Square Roots, or continual means between 10 and 1, as in the following Tablet. *Note* that the three last means marked H, I, K, are either of them mixt Numbers less then 2, and greater then 1, and have seven Cyphers placed before the significant Figures, the Logarithms in these Tables consisting only of seven places, but if you intend the Tables to consist of Eight, Ten, Twelve, Fifteen, or any Higher Number of places, you must produce so many continual meanes till the last of them have as many Cyphers before the significant Figures as the Logarithms of your intended Tables shall consist of places.

	Sq. R. or Geom. Means	Logarithms.	
	10 . 000000 &c.	1 . 0000000000000000	
A	3 . 162277660168388	0 . 5000000000000000	
B	1 . 778279410038922	0 . 2500000000000000	
C	1 . 333521432163324	0 . 1250000000000000	
	1 . 154781984689458	0 . 0625000000000000	
	1 . 074607328321318	0 . 0312500000000000	
	1 . 036632928437695	0 . 0156250000000000	
	1 . 018151721718188	0 . 0078125000000000	
	1 . 009035044841443	0 . 0039062500000000	
	1 . 004507364254462	0 . 0019531250000000	
	1 . 002251148292915	0 . 0009765625000000	
	1 . 001124941399885	0 . 0004882812500000	
	1 . 000562312602210	0 . 0002441406250000	
	1 . 000281116787782	0 . 0001220703125000	
	1 . 000140548516954	0 . 0000610351562500	
	1 . 000070271789410	0 . 0000305175781250	
	1 . 000035135277460	0 . 0000152587890625	
	1 . 000017567484423	0 . 0000076293945312	
	1 . 000008783703630	0 . 0000038146972656	
	1 . 000004391842170	0 . 0000019073486328	
	1 . 000002195918671	0 . 0000009536743164	
	1 . 000001097958730	0 . 0000004768371582	
	1 . 000000548979210	0 . 0000002384185791	
	1 . 000000274489570	0 . 0000001192092896	
	1 . 000000137244770	0 . 0000000596046448	
H	1 . 000000068612390	0 . 0000000298023224	D
I	1 . 000000034311190	0 . 0000000149011612	E
K	1 . 000000017155590	0 . 0000000074505806	G

II. Find the Logarithms correspondent to all the continual means between 10 and 1, thus ———

The Number A. being the Sq. R. of 10 divide the Logar. of 10 by 2, and the Quote is the Logar. of A. *(according to §. IV. Rule 7.)* Again the Logar. of A divided by 2, Quotes the Logar. of B; and the Logar. of B. divided by 2 quotes the Logar. of C. and so proceeding thus, you may easily find all the rest of the Logarithms, as you will see them placed in the second column of the preceding Tablet.

Note that when a mixt Number which being less than 2, and greater then 1, comes so neer unto 1 that it hath seven Cyphers before the significant Figures, the seven significant Figures of such a Number are double the seaven significant Figures of its Sq. Root.

Let the Number given be 1.0000006862238, I say as the Logar. D. 29803322 is double the Logar. E. 14901161 so is 6862238 the seaven significant Figures of A, double to 3431119 the seven significant Figures of I;

And this observation will hold true in all other Numbers of the same kind, for as many Cyphers as are placed before the significant Figures so many of their significant Figures are double to as many of the significant Figure of their Square Roots.

From hence will follow these two Corollaries.

1. That the significant Figures of the Sq. R. of a Number of this kind and the significant Figures of its Sq. R. decrease like their Logarithms *viz.* by Halves.

For as the Logar. E is half the Logar. D; so are 3431119 the significant Figures of I; Half of 6862238 the significant Figures of H.

2. That any Two Numbers of this kind being given their Logarithms, and the significant Figures of their Sq. Roots are proportional.

for as H to I. so is D. to E; or as H. to K. so is D. to G.

And this Rule will hold good in all other numbers of the same kind tho it be not one of the means between 10 and 1 as

fore

for the ſignificant Figures of the Sq. R. of any number of this kind bear the ſame proportion to his proper Logar. as the ſignificant Figures of any of the Numbers marked H, I, K. bears to his.

III. Any Number whatſoever being given to make its Logar.

You muſt produce ſo many continual means between the given Number and 1, till that which comes neereſt 1 may have ſeaven Cyphers before the ſignificant Figures; then you may find the Logar. of that mean by the Rule of Three Direct. for ———

As the ſignificant Figures of any of the Numbers marked H, I, K, are to his Logarithm.

So are the ſignificant Figures of the Sq. R. of any given Number to his proper Logarithm.

And by doubling and redoubling that Logar. according to the Number of extractions you will at laſt fall upon the Logar. required.

Let the given Number be 2, *what is the Logarithm?*

Here after Twenty four continued extractions you will find the Sq. Root to be 1.0000060443 1479 then ſay——

As 3431119, is to 1490 161, ſo is 4131479; to 17942785; which doubled and redoubled four and twenty times according to the Number of the continual meanes at laſt you will fall upon the Logar. 30102998797 5168 which is the Logar. of 2. ſee a Synopſis of the whole worke in the following Tablet.

Note that the Logar. of ſome few of the Original Numbers being thus found the Logar. of many other derivatives may be gained without the trouble of ſo many continued extractions.

Exa. Having the Logar. of 2, you may readily find the Logar. of 5, for dividing 10 by 2 the Quote is 5; but the Sum of the Logar. of the Diviſor or Quote is equal to the Logar. of the Dividend, (*by* §. 4. *Rule* 4,) Subduct therefore 0,301030 the Logar. of 2 from 1,000000 the Logar. of 10 the Remainder 0,698970 is the Logar. of 5.

Square Roots	Logarithms.
2.000000 &c.	0.301029987975168
1.414213562373909	0.150514993987584
1.189207115002721	0.075257496993792
1.090507732665257	0.037628748496896
1.044273782427413	0.018814374248448
1.021897148654111	0.009407187124224
1.010889286051700	0.004703593562112
1.005429901112800	0.002351796781056
1.002711275050200	0.001175898390528
1.001354719892100	0.000587949195264
1.000677130693060	0.000293974597632
1.000338508052670	0.000146987298816
1.000169239705290	0.000073493649408
1.000084616272680	0.000036746824704
1.000042307241380	0.000018373412352
1.000021153396950	0.000009186706176
1.000010576642540	0.000004593353088
1.000005288307280	0.000002296676544
1.000002644150140	0.000001148338272
1.000001322074190	0.000000574169136
1.000000661036870	0.000000287084568
1.000000330518780	0.000000143542284
1.000000165259170	0.000000071771142
1.000000082629580	0.000000035885571
1.000000041314790	0.000000017942785

Or you may find the Logar. of any other Number that is made by the Multiplication, or Division of these three Numbers 1, 5, and 10 *viz.* of 4, 8, 16, 32, 64, 128 &c. of 25, 125, 625, &c. of 20, 50, 100, 200 &c. following the Directions of the 3, 4, *and* 7 *Rules of* §. VI.

CHAP. V.

Of Four General Directions comprehending the Principal Business of the Rideing Officer or Country Gauger.

Direct. I.

You are to enter into your Survey Book the Names, Habitations, and Signes of every Brewer, Distiller, or other person lyable to the Duty of Excise in your Division, as also the Dimentions, and Contents of all their Tunning Vessels, according to the *Exam. Page* 104, distinguishing them by their respective Numbers of 1. 2. 3. &c. set on the Head, or some visible part of the Vessel with a Marking Iron.

To find the Whole Content of a Cask, whether Cylindrical, Spheroidal. &c.

1. *If the Cask be Cylindrical.*

Enter the Table of Cylinders with the Diameter at Top, and Length in the Side, and in the proper column stands the content in Ale Gallons and Tenths by inspection.

Exa. Suppose the Diameter 26 Inches, and Length 32 Inches. Under 26 at Top, and 32 the Length in the Side column you have 60. 2 Ale Gallons which is the Content.

2. *If the Cask be Spheroidal.*

Multiply the Difference of Diameters at Bung and Head, by 7 tenths, the Product seperating as many Figures towards the right hand as there are Decimal parts in both Multiplyers, added to the Head Diameter, gives a mean Diameter, with which and the Length enter the Table and you have the Content in A. G.

Exa. Diameter at { Bung 26 / Head 22 } Length 32 Inches.

Difference —— 4
proper Multiplier —— .7
2.8
Diameter at Head —— 22
Mean Diameter —— 24.8 under 24.75 and against 32 the Length, in the Side, in the proper Column stands 54.5 Ale Gallons &c. the Content.

And if the given Length exceeds the Length in the Table take the Number against Half which doubled is the Content.

Note that for { Much Bulging / Parabolick / Conical } Casks the Multip. is { .75 / 6 / .52 }

So that between 8 tenths and 9 tenths will all Vessels happen that are hooped, and by Practice, and experience you will soon know what Multiplyer to made use of when the Vessel lyeth before you that is to be Gauged.

§. *To take the Dimensions of a Close Cask.*

1. *For the Head:* Apply that end of the Gauging Rod where the Inches begin as close as you can to either Chine, and take the Diameters cross-wise includeing the thickness of one Chine, then if they differ take an Arithmetrical mean; and if the Heads differ you made take an Arith. mean also which is the Diameter.

2. *For the Bung Diameter:* Let the Rod descend perpendicularly from the Bung hole to the Opposite Staff, the inside of the Bung hole is the Diameter.

And if the Bung hole may not be opened extend a steight Rule or Line along the Length of the Cask just to touch the Bungs allowing for the thickness of the Hoopes, the difference between the Rule and outside of the Chine added to the Diameter at Head is the Bung Diameter. and if the Heads differ take an Arith. mean.

3. *For the Length:* Set the Length of the Chines over hanging the Heads on the Hoopes at both end, allowing an Inch and a half, or what you think fit for the thickness of the two Heads, and make two mark, or stick up two knives there, then the distance between the two marks, or knives is neer the Length of the inside of the Cask.

§. *To Gauge a Cask by the Diagonal Lines.*

Put that end of the Gauging Rod which is cut Slope-wise into the Bung hole, and thrust it home to the meeting of the Head and staves as upright as you can, observing how many Gallons appear at the middle of the Bung hole (*which you may better distinguish by marking it with Chalk as neer the middle as you can gueß*) then turn the Rod to the other end of the Cask, and thrusting it home to the Head as before, see how many

Gallons

Gallons appear, if they differ add their Semi-fference to the leſſer, or ſubduct it from the greater Number, and you have the Content.

But if it be an open Cask as a half Hogshead, or Fat. Meaſure from the middle on one ſide to the meeting of the Head and ſtaves, half the Number on the Diagonal lines is the Content either in Ale, or Wine Gallons according to the nature of the Queſtion.

§. *To Gauge a Lying Cask part full by the Table of Segments.*

To the Dry or Wet Inches given, add two Cypers, and divide it by the Bung Diameter, ſeek the Quote in the Table under V. S. and againſt it in the next column ſtands a *Segment*, which multiplyed by the whole Content the product is the Vacuity or Ullage if the Dividend were the Dry Inches, or Remaining Liquor if it were the Wet Inches, ſeperating as many Figures towards the right hand as there are places of Decimals in both Multipliers.

If the Remainder after Diviſion be above half the Diviſor add one to the Quote, if leſs then half caſt it away.

Exa. A Cask's Bung Diameter 26 Inches, and whole Content as Spheroidal 54,5 Ale Gallons; what is the Vacuity when 8 Inches are Dry?

B. D. 26) 8.00 (31 = V. S. = = = .264 Segment
78
20

Whole Content in A. G. ——— 54.5
Vacuity in A. G. ——— 14.3830

The Converſe.

B. D. 26) 18.00 (66 = V. S. = .736 Segment
156
240
234
6

Whole Content in A. G. ——— 54.5
Remaining Liquor ——— 40.1120

He that is more curious may uſe the large Table of Segments p. 52. which ſuppoſes the Diameter to be cut into 1000 equal parts; and the Area of the Circle 1,0000.

A

A Table of Segments of a Circle, whose Diameter 1.12839 *is cut by parallel Chord-lines into* 100 *equal parts, and the Area* 1.000.

V. S.	Segm.	V. S.	Segm.	V. S.	Segm.	V. S.	Segm.
.1	.002	.26	.207	.51	.513	.76	.815
.2	.005	.27	.218	.52	.525	.77	.826
.3	.009	.28	.229	.53	.538	.78	.837
.4	.013	.29	.241	.54	.551	.79	.847
.5	.019	.30	.252	.55	.564	.80	.858
.6	.025	.31	.264	.56	.576	.81	.868
.7	.031	.32	.276	.57	.589	.82	.878
.8	.037	.33	.288	.58	.601	.83	.887
.9	.045	.34	.300	.59	.614	.84	.897
.10	.052	.35	.312	.60	.626	.85	.906
.11	.060	.36	.324	.61	.639	.86	.915
.12	.068	.37	.336	.62	.651	.87	.924
.13	.076	.38	.349	.63	.664	.88	.932
.14	.085	.39	.361	.64	.676	.89	.940
.15	.094	.40	.374	.65	.688	.90	.948
.16	.103	.41	.385	.66	.700	.91	.955
.17	.113	.42	.399	.67	.712	.92	.963
.18	.122	.43	.411	.68	.724	.93	.969
.19	.132	.44	.424	.69	.736	.94	.975
.20	.142	.45	.436	.70	.748	.95	.981
.21	.153	.46	.449	.71	.759	.96	.987
.22	.163	.47	.462	.72	.771	.97	.991
.23	.174	.48	.475	.73	.782	.98	.995
.24	.185	.49	.487	.74	.793	.99	.998
.25	.196	.50	.500	.75	.804	1.00	1.000

§. *To Gauge a lying Cask part full by the Lines of Numbers, and Segments of a Circle.*

The extent from the Bung Diameter on Numbers, to the Radius 100 on Segments, will reach the same way from the Dry or Wet Inches on Numbers, to a fourth Number on Segments, which reserve;

Then by the Line of Numbers alone.

The extent from 1, to the Veſſels whole Content in Gallons, ſhall reach the ſame way from the reſerved Number, to the Gallons wanting, or in the Veſſel.

Exa. Content 54.5 *as Spheroidal.* Content——54.5

Bung Diameter 26	Bung Diameter 26
Dry Inches —— 8	Wet Inches ——18

The extent from 26 the Bung Diameter on Numbers, to 100 on Segments, will reach from 8 the Dry Inches on Numbers, to 26 on Segments, which reſerve:

Or from 18 the Wet Inches on Numbers to 76 on Segments, which reſerve.

Then the extent on Numbers from 1, to 54.5 the whole Content, will reach the ſame way from 26 the reſerved Number counted on Numbers, to 14.4 the Gallons wanting when 8 Inches are Dry:

Or from 74 the reſerved Number counted on Numbers, to 40.1 the remaining Gallons when 18 Inches are Wet.

Or thus which is the ſame in effect.

The extent from the Bung Diameter on Numbers, to the Dry or Wet Inches, will reach from the Radius 100 on Segments, to a fourth Number, which remember;

Then by the Line of Numbers alone.

The extent from 1, to the Number remembred, will reach from the whole Content, to the Gallons wanting or in the Veſſel.

Note that the Line of Segments commonly put on four feet Rules is a mean Line between a Cylinder, a Sphere, and a Spheroid, ſo will not agree with the Table of Segments.

§. *Or by a Sliding Rule.*

Set the Burg Diameter on Numbers, to the Radius 100 on Segments, and juſt againſt the Dry or Wet Inches is a fourth Number on Segments, which reſerve:

Then ſet 1 on Numbers to the whole Content, and directly againſt the reſerved Number, on the other Line of Numbers is the Ullage if the Dry Inches were given, or Remaining Liquor if the Wet Inches were given.

Or

Or thus.

Set the Bung Diameter on Numbers to the Dry or Wet Inches and just against 100 on Segments is a fourth Number, which reserve: Then set 1 on Numbers, to the reserved Number, and directly against the whole Content on the other Line of Numbers is the Gallons wanting or in the Vessel.

Direct. II.

You are to enter into your Survey Book at the Top of each respective Account, *or rather in a Book particularly for that use*, the Dimentions, and Meanes of all the Coolers, and Fats, in your Division, distinguishing likewise in every house by their respective Numbers of 1.2.3. &c. Set on some visible part of of the Vessel with a Marking Iron.

To find the Content of a Cooler, or Fat, whether Cylindrical, Conical, Eliptical, or Spheroidal.

1. Of a Cylindrical Cooler, or Fat.

Enter the Table of Cylinders with the Diameter at Top, and Depth, or Wet Inches in the Side, and in the proper Column stands the Content in Ale Gallons &c. as the Title of the Table expresseth.

2. Of a Conical Cooler, or Fat.

Add together the Diameters at Top and Bottom, the Half of which is an Arithmetical mean, and neer enough in practise being only used in small Vessels, find this mean Diameter at Top of the Table, and the Depth, or Wet Inches in the Side, and in the proper Column you have the Content in Ale Gallons &c. by inspection.

Diameter at { Top — 38 / Bottom 36 } Depth 20 Inches.

Sum — 74

Half — 37 with which enter the Table and directly under it, and against 10 the Wet Inches stands, 36.1 Gallons for the Content.

He that desires more exactneß may take an Arithmetical mean between the Diameter at the Surface of the Liquor, and Diameter at Bottom.

Or

Or follow the Directions in Chap. II. Prop. XII.

3. Of an Elliptical or Oval Cooler, or Fat.

If the Bases above and below are equal, take the Dimentions Cross-wise, and add them together the half is an Arithmetical Mean.

But if the Bases above and below are unequal, then take the Diameters both at Top and Bottom cross-wise, and adding them together the one fourth part is a mean, with which and the Depth, or Wet Inches enter the Table, and you have the Content in Ale Gallons &c.

Longest } Diam. { 40 } Depth, or Wet Inches 25
Shortest } { 32 }

Sum——72
Half——36 at Top and 25 in the Side gives 90.2 A.G.

Longest } Diameter at Top { 60 }
Shortest } { 54 }
Longest } Diameter at Bot. { 50 } Depth, or Wet Inches 20
Shortest } { 46 }

Sum——210
Fourth — 52.5 per 20 gives——153.3 A.G

Now tho these wayes are not Mathematically true yet in Vessels where the Difference of Diameters is small they may be nigh enough in Practice, being very useful and intelligible.

Those that are more curious may have recourse to the Directions in Chap. II. Prop. XIII.

4. Of a Spheroidal Cooler, or Fat, (as are Butts, Pipes, Hogsheads, &c.)

Multiply the Difference of the Diameters at Bung and Head by 7 tenths, the Product seperating as many Figures towards the right hand as there are Decimals in both Multipliers added to to the Head-Diameter gives a Mean Diameter, with which and the Depth, or Wet Inches enter the Table and you have the Content in Ale Gallons &c.

Or Multiply the Difference of the Diameters at Bung and Surface

Surface of the Liquor by 7 tenths, the Product seperating as many Figures as are required by Decimals added to the Diameter of the Surface of the Liquor gives a mean Diameter.

Then if the Cask be more then half full, with this mean Diameter, and the Wet Inches above the Bung, (*found by subducting the Semi-Length from the Wet Inches*) enter the Table and in the proper Column you have the Content of the Replete part above the Bung, which added to the Semi-Content, the Sum is the Content required.

But if the Cask be less then half full, with this mean Diamter, and the Dry Inches below the Bung, (*found by Subducting the Wet Inches from the Semi-Length*) enter the Table, and in the proper Column you have the Content of the Vacant part below the Bung, which Subducted from the Semi-content the Remainder is the Content required.

1. *Ex.* Diam at { Bung — 26 / Head — 22 } Length, or Wet Inches 24.

Difference — 4

Multip. proper — .7

2.8

Head Diameter add — 22

Mean Diameter — 24.8 under 24.75 and against 24 is 40.9

And if the Wet Inches had bin 8 the Content would be 13.6

2. *Ex.* Diamter at { Bung — 26 / Liquors Surface — 25 }

Difference — 1

proper Multiplier — .7

.7

Diameter at the surface of Liquor 25.

Mean Diameter — 25.7 under 25.75 and against 8 the Dry Inches stands 14.8 Ale Gallons &c.

Now 14.8 { added to / Sub: from } 27.3 the Semi cont. gives { 42.1 / 12.5 } A. G.

Note that by help of the Wet Inches you may find the Diamter at the Surface of the Liquor, for at your next visit when the

Vessel

Veſſel is empty, if you Set off the Wet Inches from the Bottom you may actually meaſure the Diameter required.

Or if you will take the pains you may Calculate it as directed Chap. II. Prop. XV.

When you cannot put by the Yeaſt or Barm by reaſon of its working to take the Wet Inches do thus———

Haveing alwayes ready a peice of Cork with a ſtring thro the Center, and ſo much Lead fixed to the Bottom as the Top thereof may exactly touch the Surface of the Liquor, let it down into the Fat, then let a Rule deſcend to the Bottom of the Fat in the midſt thereof, and bring the Cork to the Side of the Rule (*letting the Cork ſtill Swim upon the Surface of the Liquor*) then take them both out keeping them cloſe together and the upper part of the Cork will cut the Rule ſo much as it contains Inches in Depth.

Or you may take the Wet Inches with your Rule thro a Tin pipe.

§ *Of the Four Foot Rule and Semi-circle.*

Put the Pin in the Center of the midle joynt into the Center hole of the Semi-circle, and turn it about till the Pin in the Head Leg fit in the other hole: Then open the middle joynt like a pair of Compaſſes till the two ends of the Rule exactly touch the Veſſel at that part you intend to take Diamter, and the Inner edge of the moveing Leg of the Rule ſhall ſhew on the outermoſt Line of the Semi-circle the Diameter in Inches to 48, and on the inner Lines the Area upon 1 Inch in Ale Gallons and Tenths, or Pints, which multiplied by any Number of Wet Inches produces the whole Content.

§ *Of Gauging by a Slideing Rule.*

To do this your Rule muſt have on one Side a Single Line of Numbers, and oppoſite againſt it on the ſame Face a Double Line of Numbers, with ſmal braſs Pins to ſhew the places of the Gauge Points: Set the Gauge Point on the Single Line, always to the Depth or Length on the Double line, and directly againſt the mean Diameter on the Single line is the Content on the Double line.

How

How to find a Diameter you are ſufficiently inſtructed.

Exa. Let the Depth be 10, and the Mean Diameter 37 Inches. Set 18. 95 the Gauge Point for an Ale Gallon, to 10 the Depth on the Double line, and juſt againſt 37 the mean Diameter on the Single line, ſtands 38. 1 Ale Gallons the Content on the Double line.

§ *Of Gauging by the Line of Numbers.*

The Extent from the Gauge Point to the mean Diameter, being twice repeated the ſame way from the Length or Depth gives the Content.

Exa. Let the Depth be 10, and the Mean Diameter 37 Inches as before.

The extent from the Gauge Point for an Ale Gallon to 37 the mean Diameter, ſhall reach the ſame way at twice turning the Compaſſes from 10 the Depth, to 38. 1 the Content in Ale Gallons and Tenths.

Direct. III.

You are upon your firſt Survey to take an exact Account of what Stock you find in your Diviſion, and enter it under the reſpective Names of the Perſons, expreſſing the day of the moneth when you took the ſame, and ſo twice or thrice in every week regularly inſerting againſt the day of the month the Increaſe or Decreaſe of each reſpective Veſſel, number of Wet Inches, and charge when caſt up according to the following Method: Which Method was firſt propoſed by the ingenious Mr. *Chriſt. Banſon* and ſince it hath received ſome improvements which are found very uſefull in Practice.

Table

William Dixon of *Ripon*,
at the Sign of the *Fox*.

						B.	B.	Sy.	Sy.		
		1	2	3	4	5	6	7	8	9	10
° N	Br.	1683.	+ 25.5 54	27 22 36	25 20 28	20 19 20	✕ 26 54	+ 25.5 54	+ 25.8 54	+ 25.5 54	10 10
		May. 1	0	1	1	1	1	1	1	1	00
	1	4	7	1 d	0	0	1 d	1	1 d	1	00
	2	6	10	1	0	0	1 d	1	1 d	1	00
	3	8	14	3/4	6	1	1 d	1	1 d	1	00
		10	20	0	16	1	3/4	1	1 d	1	00
4.	5	15	0	1	6	3/4	3/4	1	1 d	1	10
		17	0	1	10	0	1/4	1	3/4	1	00
	6	19	6	1 d	0	0	1/2	1	1/2	1	00
	7	22	6	1/2	1	0	1/2	1	1/2	1	10
		25	12	0	1	0	1/4	1	1/4	1	10
	8	28	20	1	1	0	1/4	1	1/4	1	00
	9	*June.* 1	6	1	1/2	0	1/8	1	1/8	1	00

TABLES,
FOR THE
GAUGING
OF
BREWERS TUNS
and
CASKS.

By *William Hunt* Practical Gauger.

YORK.
Printed, by *John White*, for the
AUTHOR 1683.